THE ACADEMY OF ST NICHOLAS	**Y12 Pure**	**CH14** 14.1,14.2,14.3,14.4, 14.5,14.6,14.7,14.8	**Exponentials and logarithms**	**Lessons 5**

Essential Knowledge Milestones

- know and be able to use the function $a^x$and its graph, where *a* is positive;
- know and be able to use the function e^x and its graph;
- know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications;
- know and be able to use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$;
- know and be able to use the function $\ln x$ and its graph;
- know and be able to use ln x as the inverse function of e^x ;
- understand and use the laws of logarithms:

  $\log_a x + \log_a y = \log_a (xy)$

  $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

  $k\log_a x = \log_a x^k$(including, for example, k =–1 and $k = -\frac{1}{2}$)
- be able to solve equations of the form $a^x = b$;
- be able to use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y;
- understand and be able to use exponential growth and decay in modelling, giving consideration to limitations and refinements of exponential models.

Assumed Prior Knowledge/ Links / Interleaving

- Surds and Indices: the laws of indices lead directly to the laws of logarithms
- Differentiation: to appreciate the gradient function of $y = e^{kx}$
- Graphs and transformations: to appreciate the link between $y = e^x$ and $y = e^{kx}$
- Geometric sequences: taking the logarithm of each term in a geometric sequence results in an arithmetic sequence
- Integration: $\int \frac{1}{x} dx = ln\, x + c$
- Differential equations: the general solution of $\frac{dy}{dx} = y$ is $y = A\, e^x$

Teaching Points

- When sketching the graph of a^xstudents should understand the difference in shape between *a* < 1 and *a* > 1.
- Explain to students that e^x is a special case of a^x. Graphs of the function e^x should include those in the form $y = e^{ax+b} + c$.
- Students should realise that when the rate of change is proportional to the *y*-value, an exponential model should be used.
- An ability to solve equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.
- Students can use the laws of indices to prove the laws of logarithms and show that $\log_a a = 1$.
- In solving equations students may use the change of base formula. Solving equations questions may be in the form $2^{3x-1} = 3$.
- Students should be able to plot log *y* against log *x* and obtain a straight line where the intercept is $\log_a$ and the gradient is *n* and plot log *y* against *x* to obtain a straight line where the intercept is log *k* and the gradient is log *b*. There should be discussion about why this is an appropriate model and why it is only an estimate.
- Contexts for modelling should could include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth. Students should be familiar with terms such as initial, meaning when *t* = 0. They may need to explore the behaviour for large values of *t* or to consider whether the range of values predicted is appropriate. Consideration of a second improved model may be required.

Potential Barriers to Access /Misconceptions

- Errors seen in exam questions where students have to sketch exponential curves include: stopping the curve at x = 0; getting the wrong *y*-intercept; and believing the curve levels off to *y* = 1 for x < 0.
- When using laws of logs to answer proof or 'show that' questions, students must show all the steps clearly and not have jumps in their working out.
- Incorrectly stating the laws of logarithms; for example $log_{10}\, a - log_{10}\, b = \frac{log_{10}\, a}{log_{10}\, b}$
- Solving equations with products where logarithms need to be taken. E.g. $P = aT^n$ $\Rightarrow$ $log\, P = log\, a \times n\, log\, T$ instead of $log\, P = log\, a + n\, log\, T$
- After reducing to linear form, linking gradients and y -intercepts from their straight line to the original equation.

Opportunities for Reasoning/Problem Solving/Proofs

- Students can look at different models for population growth using the exponential function.
- Use graphing software to investigate varying the parameters of a population model.

	Questions & Prompts
	• Is taking logs a useful technique for solving the equation $3^x = x^3$? • Explain how can you use the fact that $2^{10} \approx 1000$ and $3^3 \approx 5^2$ to find approximate values of $log_{10} 2$, $log_{10} 5$ and $log_{10} 3$? • How can you use the value of $log_{10} 2$ to determine the number of digits in the number 2^{74}?

Key Mathematical Vocabulary	Exponential, exponent, power, logarithm, base, initial, rate of change, compound interest

Personal Development	Notes	Resources
Pupils are taught that's "patience" is an asset, when approaching mathematical problems, and resilience is needed to preserve to complete a task.		