THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH13 13.1,13.2,13.3,13.4, 13.5,13.6,13.7	Integration	Lessons 5

Essential Knowledge Milestones

- know and be able to use the Fundamental Theorem of Calculus;
- be able to integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples.
- be able to evaluate definite integrals;
- be able to use a definite integral to find the area under a curve.

Assumed Prior Knowledge/ Links / Interleaving

- Algebraic manipulation
- Differentiation
- AS differentiation: as with all inverse processes, fluency with the standard process helps with the reverse
- Numerical methods: the trapezium rule is used to find an approximation to the area of a region under a curve

Potential Barriers to Access /Misconceptions

- Students sometimes have difficulty when integrating expressions involving negative indices.
- Forgetting to add + c when working out indefinite integrals is also a very common mistake.
- Lack of algebraic fluency can cause problems for some students, particularly when negative/fractional indices are involved or when a negative number is raised to a power. Arithmetic slips are also a common cause of lost marks, often when negative numbers are substituted and subtracted after integration.
- Students are generally more successful if they expand any brackets before attempting to integrate the function.
- Mixing up the rules for differentiation and integration
- Omitting the constant of integration, especially when finding the original equation of the curve from the given derivative.
- Dealing with denominators; e.g. $\int \frac{6}{x^3} dx$ as $x^{\frac{6}{3}}$ or $\frac{6x}{x^4}$.
- Integrating a constant a as $\frac{a^2}{2}$

Teaching Points

- Integration can be introduced as the reverse process of differentiation. Students need to know that for indefinite integrals a constant of integration is required.
- Similarly to differentiation, students should be confident with algebraic manipulation. For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected.
  Introduce students to the integral sign; this can be useful in setting work out clearly on these sorts of questions and will be used later in definite integration.
- Given f′(x) and a point on the curve, students should be able to find an equation of the curve in the form y = f(x).
- It is important that students show their working out clearly as mistakes are easily made when putting values into a calculator. Students should also be encouraged to check their answers. Calculators that perform numerical integration can be used as a check, but a full method will be needed.
- Students will be expected to understand the implication of a negative answer from indefinite integration.
- Links can be made with curve sketching in questions where students need to find the points of intersection with the x-axis for a curve in order to find the limits of integration.
- Areas can be made up of a combination of a curve and a line so further links can be made to coordinate geometry.

Opportunities for Reasoning/Problem Solving/Proofs

- Students should be able to explain the need for the + c in indefinite integration.
- Discuss the implication of a negative answer to encourage students reasoning skills. (For Area)
- Prove that $\int_0^a x^n \, dx = -\int_{-a}^0 x^n \, dx$ for odd values of n.
- Prove the Fundamental Theorem of Calculus: i.e. that the reverse process of differentiation gives the area under a curve

Questions & Prompts

- Give me an example of a curve for which $\int_{-2}^0 y dx = -\int_0^2 y dx$
- $\int_0^2 1 - x \ \ dx = 0$. Make up a similar example.

Key Mathematical Vocabulary	Calculus, differentiate, integrate, reverse, indefinite, definite, constant, evaluate, intersection.

Personal Development	Notes	Resources
Pupils to show integrity and respect each other's opinion when peer marking each other's work against mark schemes		