THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH01 12.1,12.2,12.3,12.4, 12.5,12.6,12.7,12.8,12.9,12.10,12.11	Differentiation	Lessons 5

Essential Knowledge Milestones

- understand and be able to use the derivative of f(x) as the gradient of the tangent to the graph of *y* = f(x) at a general point (x, *y*);
- understand the gradient of the tangent as a limit and its interpretation as a rate of change;
- be able to sketch the gradient function for a given curve;
- be able to find second derivatives;
- understand differentiation from first principles for small positive integer powers of *x*;
- be able to differentiate x^n, for rational values of *n*, and related constant multiples, sums and differences.
- be able to apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points;
- be able to identify where functions are increasing or decreasing.

Assumed Prior Knowledge/ Links / Interleaving

- Indices: Students need to be fluent in writing expressions such as $\sqrt{x^3}$ in index form
- Equations of a straight line: knowing how to find the equation of a tangent or normal given a point on a curve and the gradient at that point
- Completing the square: another way to find turning points of quadratics
- Repeated roots: A tangent to a curve can be found using the fact that there are repeated roots when solving simultaneously
- Function notation
- Area of 2D shapes
- Volume and surface and of 3D shapes

Teaching Points

- Students should know that $\frac{dy}{dx}$ is the rate of change of *y* with respect to *x*.
- Knowledge of the chain rule is not required.
- The notation f′(x) may be used for the first derivative and f″(x) may be used for the second order derivative.
- Students should be able to identify maximum and minimum points as points where the gradient is zero.
- Cover the use of the second derivative to establish the nature of a turning point.
- Students should be able to sketch the gradient function f′(x) for a given curve *y* = f(x), using given axes and scale. This could involve speed and acceleration for example.
- Students should know how to differentiate from first principles. Students should be able to use, for *n* =2 and *n* = 3, the gradient expression $\lim_{h\to 0}\left(\frac{(x+h)^n-x^n}{h}\right)$. The alternative notations *h* → 0 rather than $\delta x \to 0$ are acceptable.
- Students will need to be confident in algebraic manipulation of functions to ensure that they are in a suitable format for differentiation. For example, students will be expected to be able to differentiate expressions such as (2*x* + 5)(x −1) and $\frac{x^2+3x-5}{4x^{\frac{1}{2}}}$, for x > 0. Mistakes are easily made with negative and/or fractional indices so there should be plenty of practice with this.
- Students should be able to use differentiation to find equations of tangents and normals at specific points on a curve. This reviews and extends the earlier work on coordinate geometry.
- Maxima, minima and stationary points can be used in curve sketching. Problems may be set in the context of a practical problem. This could bring in area and volume from GCSE (9-1) Mathematics as well as using trigonometry.
- Students will need plenty of practice at setting up equations from a given context, in some cases this may include showing that it can be written in a particular form. Where students are given the answer to work towards they must be aware that they need to work forwards showing all steps clearly rather than starting with the answer and working backwards.
- Students need to know how to identify when functions are increasing or decreasing. For example, given that f ′(x) = $x^2 - 2 + \frac{1}{x^2}$, , prove that f(x) is an increasing function.
- Use graph plotting software that allows the derivative to be plotted so that students can see the relationship between a function and its derivative graphically.

Potential Barriers to Access /Misconceptions	**Opportunities for Reasoning/Problem Solving/Proofs**
• Algebraic manipulation, particularly where surds are involved, can cause problems for students. For example, when multiplying out brackets and faced with $-4\sqrt{x} \times -4\sqrt{x}$ common incorrect answers are $-4\sqrt{x}$, $\pm16\sqrt{x}$ and $\pm16x^{\frac{1}{4}}$. Similarly, when dividing by $\sqrt{x}$, some students think that $\frac{x}{\sqrt{x}}$ = 1. • Students may have difficult differentiating fractional terms such as $\frac{8}{x}$ if they are unable to rewrite this as $8x^{-1}$ before differentiating. • When working out the equations of tangents and normal, some students mix the gradients and equations up and end up substituting in the wrong place. • Questions involving finding a maximum or minimum point do require the use of calculus and attempts using trial and improvement will receive no marks. • When finding a stationary point, some students use inequalities as their condition rather than equating their derivative to zero. Another error is to differentiate twice and solve f''(x) = 0. • When applying differentiation in context, students should be ensure they give full answers and not just a partial solution. For example if asked to find the volume of a box they must not stop after finding the side length. • Assuming that at a point where $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ there must be a point of inflection • Not appreciating differentiation from first principles, especially when the question guides the student through it. • Seeing the link from $\frac{dy}{dx}$, via gradients, to equations of tangents and normals. •	• Maxima and minima problems set in the context of a practical problem, e.g. minimising the materials required to make a container of a particular shape. The open box problem – simple cases and the general case. • Differentiation can be linked to many real-world applications, there can be discussion with students about contexts and the validity of solutions. **Questions & Prompts** • How would you explain the role of chords in differentiation from first principles? • If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ at $(-1,2)$a full explanation of why there is a point of inflection at $(-1,2)$ on the curve $y = x^3 + 3x^2 + 3x + 3$? • Give me an example of a curve with a maximum point at $(-2,-2)$ • P lies on the curve $y = 6 - 3x^2$. At P the gradient is 6, what is the x coordinate of P? • $f'(x) = (x-3)(2-x)$, determine the value for which *f* is increasing. • The height of a ball, h meters, above the ground, t seconds after being thrown vertically upwards is given by $h = 5t - 2t^2$. Find the maximum height of the ball. • Charlie got to this stage of his maths work, $\frac{dy}{dx} = 2x - 4$, he substitutes x=4 into the equation. What does this tell him? • Find the location and nature of the turning point for the equation $y = x^2 + 2x + 4$

Key Mathematical Vocabulary	Differentiation, derivative, first principles, rate of change, rational, constant, tangent, normal, increasing, decreasing, stationary point, maximum, minimum, integer, calculus, function, parallel, perpendicular.

Personal Development	**Notes**	**Resources**
Pupils are taught that they must be able to put into 'action', from feedback provided by their teacher. They will learn to strive for their 'personal best' in the quality of their outcomes, in order to successfully meet criteria.		