THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH08 8.1,8.2,8.3,8.4,8.5	Binomial Expansion	Lessons 3

Essential Knowledge Milestones

- understand and be able to use the binomial expansion of $(a + bx)^n$ for positive integer n;
- be able to find an unknown coefficient of a binomial expansion.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: Expanding brackets
- Surds and indices: for example, expanding $(1+\sqrt{2})^n$
- Logarithms and exponentials: expanding $\left(1+\frac{1}{n}\right)^n$ for large values of $n$approaches a limit (the exponential number).

Teaching Points

- Students should initially be introduced to Pascal's triangle, which can be used to expand simple brackets.
- Students will need to be familiar with factorials and the ${}_nC_r$ notation.
- Introduce the formal binomial expansion in the same way as the formula booklet and discuss the various terms to ensure all students understand.
- Setting out work clearly and logically will be invaluable in helping students to achieve the final answer and also to spot mistakes if necessary.
- Where there is a coefficient of x (other than 1) students will need to be reminded that the power applies to the whole term, not just the x, and that answers must be simplified appropriately. Negative and fractional coefficients will also need practice.
- The limitations of the binomial expansion should be discussed.
- Students should practice finding the coefficient of a single term, they should also be able to deal with setting up simple algebraic equations to find unknown constants.

Use of the binomial expansion can be linked to basic probability and approximations

Potential Barriers to Access /Misconceptions

- Marks are most commonly lost in exam questions because of errors in expanding terms. For example, not including the coefficient when calculating, say, $(ax)^2$; not simplifying terms fully; sign errors; and omitting brackets. Good notation will help to avoid many of these mistakes.
- When writing expansions which involve unknown constants, some students fail to also include the x's in their expansion.
- When using their expansions to work out the value of a constant, a significant number of students do not understand that the coefficient does not include the x or x^2 part and so are often unable to form an equation in the unknown alone.
- Questions often go on to ask students to use their binomial expansion to evaluate a number raised to a power. For example, evaluating $(1.025)^8$ by substituting x = 0.025 into an expansion for $(1 + x)^8$. Students should be advised that simply using their calculator to evaluate $(1.025)^8$ will gain no marks as it is not answering the question.
- Raising only part of the term to the appropriate power. For example, in $(1-2x)^6$, giving the third term as ${}^6C_3x^3$ or ${}^6C_3(2x)^3$ rather than ${}^6C_3(-2x)^3$
- Bracketing errors when evaluating a binomial coefficient e.g. giving the x^3 term in $(3-2x)^5$ as $10 \times 3^2 \times 2x^3$ or even $10 \times 3 \times (-2) \times x^3$.
- Wasting time writing out the full expansion instead of finding the coefficient of the required term.

Opportunities for Reasoning/Problem Solving/Proofs

- Students can be encouraged to discover the link between Pascal's triangle and the expansion of simple brackets.
- Students could look at find the general term of a particular expansion.

Questions & Prompts

- How would you explain why the coefficient of x^7 in $(2+x)^{10}$ is ${}^{10}C_7 \times 2^3$?
- Change one number in $(1+1x)^4$ so that the coefficient of x in the expansion is 32.
- Give me two examples of binomial expansions in which all the coefficients are odd.
- What is the value of ${}^{45}C_{45}$?
- What is the coefficient of the x^3y^2 term in the expansion $(x - 3y)(x + y)^4$

Key Mathematical Vocabulary	Binomial, coefficient, deduction, polynomials, factorisation, quadratic, cubic, quartic, expand,

Personal Development	**Notes**	**Resources**
Pupils are taught to be able to identify a situation whereby a particular maths skill is applied to a problem solve a question, and to have belief in their own ability.		