THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH07 7.1,7.2,7.3,7.4,7.5	Algebraic Methods Algebraic fractions, Dividing polynomials, Factor Theorem, Proofs	Lessons 4

Essential Knowledge Milestones

- be able to use algebraic division;
- know and be able to apply the factor theorem;
- be able to fully factorise a cubic expression;
- understand and be able to use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion;
- be able to use methods of proof, including proof by deduction, proof by exhaustion and disproof by counter-example.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: Factorising quadratics, notation, Expanding brackets, Substitution
- Proof will also be included in later topics.

Potential Barriers to Access/Misconceptions

- The majority of errors seen in exam questions are not due to misunderstanding the method, but instead arithmetic and algebraic mistakes. For example, incorrect simplification of terms – especially those involving fractions; mistakes with negative numbers; and writing expressions rather than equations.
- Students should be aware that long division is not always the best or quickest method to use and sometimes results in some complicated algebra.
- When using the factor theorem, stress the importance of checking the value that is substituted; a common error is to use, for example, f(1) rather than f(–1).
- You should also emphasise the importance of fully factorising expressions, as a fairly significant number of students stop when they have reached one linear factor and a quadratic factor.
- In questions which specify 'by using the factor theorem' attempting to answer by other methods, such as polynomial division
- Division by $x + a$ when a is the known root
- Errors with signs when carrying out polynomial division

Teaching Points

- When using algebraic division, only division by ($ax + b$) or ($ax – b$) will be required.
- Different methods for algebraic division should be considered depending on students' prior experience and preferred ways of working. Whichever method is used, clear working out should be shown.
- Equations in which the coefficient of x or x^2 is 0 for example $x^3 + 3x^2 - 4$ or $2x^3 + 5x - 20$ will need additional explanation and practice.
- Students should know that if f(x) = 0 when x = a, then (x – a) is a factor of f(x). Questions in the form ($ax + b$) should be covered.
- Where a negative is being substituted into the equation the distinction between $(-2)^2$ and -2^2 will be important especially when students are using a calculator as examiners often comment on the fact that students will sometimes evaluate $(-2)^2$ as -4.
- Factor theorem can be used to find an unknown constant. For example: Find a given that $(x - 2)$ is a factor of $x^3 + ax^2 - 4x + 6$. Two conditions can also be given in order to form simultaneous equations to solve.
- When fully factorising a cubic, emphasis should be placed on choosing appropriate values. The final answer may need to be written as a factorised cubic or, alternatively, as the solutions to an equation which can then be used to sketch the curve. Students sometimes use the roots of a polynomial equation to help them factorise but this method must be used with care. Questions sometimes use the word 'hence' and so students must be careful which method they chose in these cases.
- This is an excellent opportunity to review curve sketching by asking students to give a sketch following factorisation.
- Students should be familiar with basic proofs from GCSE (9-1) Mathematics this knowledge can be built upon to look at the different types of proof. Students will need to understand how to set out each type of proof; the correct conventions in language and layout should be encouraged.

Opportunities for Reasoning/Problem Solving/Proofs

- The factor theorem can be introduced through investigation by substituting different values and checking against division to look for patterns.
- Proof gives the opportunity to review previous concepts in a different way for example coordinate geometry.

	Questions & Prompts
	• How would you explain how to divide $2x^3 - 5x^2 + 3x - 2$ by $x - 2$? • What is the same and what is different about the factor theorem and dividing polynomials? • Given that (x + 2) is a factor of $x^3 + px^2 + 2x - 8$, what is the value of p?

Key Mathematical Vocabulary	proof, assumptions, deduction, exhaustion, disproof, counter-example, polynomials, factorisation, quadratic, cubic, quartic, conjecture, prediction, rational number, implies, necessary, sufficient, converse, fully factorise, factor, expand, therefore, conclusion

Personal Development	Notes	Resources
Pupils are taught to be honest and 'truthful' in the judgments they make when they self-assess their learning as it serves to aid the teacher in planning for future learning or revisiting material for overlearning & embedding.		