THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH03 3.4,3.5,3.6,3.7	Equations & Inequalities Inequalities: Linear & Quadratic	Lessons 4

Essential Knowledge Milestones

- be able to solve linear simultaneous equations using elimination and substitution;
- be able to use substitution to solve simultaneous equations where one equation is linear and the other quadratic.
- know how to express solutions through correct use of 'and' and 'or' or through set notation;
- be able to interpret linear and quadratic inequalities graphically;
- be able to represent linear and quadratic inequalities graphically.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: Simultaneous equations
- Ensure simultaneous equations solution can be found on calcs
- intersection of lines and circles
- Graphical representation of equation and understanding that the insertions are the point of simultaneous equations

Potential Barriers to Access /Misconceptions

- Students may make mistakes when multiplying or dividing inequalities by negative numbers.
- In exam questions, some students stop when they have worked out the critical values rather than going on to identify the appropriate regions. Sketches are often helpful at this stage for working out the required region.
- It is quite common, when asked to solve an inequality such as $2x^2 – 17x + 36 < 0$ to see an incorrect solution such as $2x^2 – 17x + 36 < 0 \Rightarrow (2x – 9)(x – 4) < 0 \Rightarrow x < \frac{9}{2}, x < 4$.
- If x is between -2 and -5 writing the inequality as $-2 < x < -5$ rather than $-5 < x < -2$
- Trying to combine two separate inequalities as a single expression, for example writing $x < -4$ and $x > 3$as $-4 > x > 3$
- Thinking that $(x-4)(x+2) > 0$ means $(x-4) > 0$ and $(x+2) > 0$
- Believing that $(x+3)^2 < 16$ has the same set of solutions as $(x+3) < 4$
- When rationalising a denominator, failing to divide both terms in the numerator by the result in the denominator. E.g. $\frac{7+4\sqrt{3}}{4} = \frac{7}{4} + 4\sqrt{3}$ or $7 + \sqrt{3}$ instead of $\frac{7}{4} + \sqrt{3}$
- Often cancelling inside the square root by a denominator rather than its square; e.g. $\frac{\sqrt{20x^2}}{2} = \sqrt{10}x$ or even $10x$ instead of $\sqrt{5}x$

Teaching Points

- Provide students with plenty of practice at expressing solutions in different forms using the correct notation. Students must be able to express solutions using 'and' and 'or' appropriately, or by using set notation. So, for example:
    - $x < a$ **or** $x > b$ is equivalent to $\{x: x < a\} \cup \{x: x > b\}$
    - and $\{x: c < x\} \cap \{x: x < d\}$ is equivalent to $x > c$ **and** $x < d$.
- Inequalities may contain brackets and fractions, but these will be reducible to linear or quadratic inequalities. For example, $\frac{a}{x} < b$ becomes $ax < bx^2$.
- Students' attention should be drawn to the effect of multiplying or dividing by a negative value, this must also be taken into consideration when multiplying or dividing by an unknown constant.
- Sketches are the most commonly used method for identifying the correct regions for quadratic inequalities, though other methods may be used. Whatever their method, students should be encouraged to make clear how they obtained their answer.
- Students will need to be confident interpreting and sketching both linear and quadratic graphs in order to use them in the context of inequalities.
- Make sure that students are also able to interpret combined inequalities. For example, solving
    - $ax + b > cx + d$
    - $px^2 + qx + r \geq 0$
    - $px^2 + qx + r < ax + b$
    - and interpreting the third inequality as the range of x for which the curve $y = px^2 + qx + r$ is below the line with equation $y = ax + b$.
- When representing inequalities graphically, shading and correctly using the conventions of dotted and solid lines is required. Students using graphical calculators or computer graphing software will need to ensure they understand any differences between the conventions required and those used by their graphical calculator.

Opportunities for Reasoning/Problem Solving/Proofs

- Financial or material constraints within business contexts can provide situations for using inequalities in modelling. For those doing further maths this will link to linear programming.
- Inequalities can be linked to length, area and volume where side lengths are given as algebraic expressions and a maximum or minimum is given.
- Following on from using a quadratic graph to model the path of an object being thrown, inequalities could be used to find the time for which the object is above a certain height.
- Prove that multiplying an inequality throughout by -1 is the same as reversing the sign.

	Questions & Prompts
	- What questions could be used to tease out why multiplying an inequality by a negative number means you have to reverse the inequality? - When $x = \frac{1}{3}$, $x^3 < x^2 < x < 1 - x < \frac{1}{x}$. What happens to the order of inequalities for other values of x?

Key Mathematical Vocabulary	sketch, plot, quadratic, region,

Personal Development	Notes	Resources
Pupils are taught that they must show ‘compassion’ when working collaboratively as peer support may be required by those potentially making mistakes.		