THE ACADEMY OF ST NICHOLAS	**Y12 Pure**	**CH01** 1.1,1.2,1.3,1.4, 1.5,1.6	**Algebraic Expressions: Surds, Indices & Brackets**	**Lessons 4**

Essential Knowledge Milestones

- be able to perform essential algebraic manipulations, such as expanding double brackets, collecting like terms, factorising etc;
- understand and be able to use the laws of indices for all rational exponents;
- be able to use and manipulate surds, including rationalising the denominator

Assumed Prior Knowledge/ Links / Interleaving

- Logarithms: logs allow us, for example, to make 3 the subject of the equation $2^3 = 8$
- Quadratics: the exact roots of a quadratic often require surd form
- Expanding double brackets – grid or "smiley face"

Teaching Points

- Emphasise that in many cases, only a fraction or surd can express the exact answer, so it is important to be able to calculate with surds.
- Ensure students understand that $\sqrt{a} + \sqrt{b}$ is not equal to $\sqrt{a+b}$ and that they know that $a^{\frac{m}{n}}$ is equivalent to $\sqrt[n]{a^m}$ and that a–m is equivalent to $\frac{1}{a^m}$.
- Most students understand the skills needed to complete these calculations but make basic errors with arithmetic leading to incorrect solutions.
- Questions involving squares, for example $(2\sqrt{3})^2$, will need practice.
- Students should be exposed to lots of simplifying questions involving fractions as this is where most marks are lost in exams.
- Recap the difference of two squares (x + y)(x – y) and link this to $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ = x – y , explaining the choice of term to rationalise the denominator.
- Provide students with plenty of practice and ensure that they check their answers.

Potential Barriers to Access /Misconceptions

- misinterpreting $\left(a\sqrt{b}\right)^2$ as $(a + \sqrt{b})^2$;
- evaluating $(\sqrt{2})^2$ as 4 instead of 2;
- Slips when multiplying out brackets; basic arithmetic errors;
- leaving surds in the denominator rather than fully simplifying fractions.
- Examples of errors with indices are, writing $\frac{1}{3x}$ as $3x^{-1}$ and writing $\frac{4}{\sqrt{x}}$ as $4x^{\frac{1}{2}}$; these have significant implications later in the course (e.g. differentiation)
- Mixing up rules such as $a^3 \times a^2 = a^6$ and $2x^{-3} = \frac{1}{2x^3}$
- When rationalising a denominator, failing to divide both terms in the numerator by the result in the denominator. E.g. $\frac{7+4\sqrt{3}}{4} = \frac{7}{4} + 4\sqrt{3}$ or $7 + \sqrt{3}$ instead of $\frac{7}{4} + \sqrt{3}$
- Often cancelling inside the square root by a denominator rather than its square; e.g. $\frac{\sqrt{20x^2}}{2} = \sqrt{10x}$ or even $10x$ instead of $\sqrt{5}x$

Opportunities for Reasoning/Problem Solving/Proofs

- Include examples which involve calculating areas of shapes with side lengths expressed as surds. Exact solutions for Pythagoras questions is another place where surds occur naturally.
- Prove that any irrational number can be a root of at most one cubic equation of the form $x^3 + ax = b$ where a and b are rational.
- Prove that$\sqrt{2}$ is irrational

Questions & Prompts

- Give me an example of a number that is equal to $3\sqrt{2}$ …and another….and another
- Change one number in $(2 + \sqrt{8})(4 - \sqrt{2})$ so that the product is a rational number.
- $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$. Always true, sometimes true, never true?
- Give me an example of a number between $5\sqrt{6}$ and $6\sqrt{5}$.
- Express $\sqrt{75} + \sqrt{27}$ in the form $a\sqrt{b}$ where a and b are integers and b is as mall as possible

Key Mathematical Vocabulary	Denominator, expression, index, linear, identity, factorise, power, rational, irrational, reciprocal, root, surd, rationalise, exact

Personal Development	Notes	Resources
Pupils are taught that they must 'respect' each other's opinions and well-being when working collectively in class. Pupils to learn that mathematicians have 'ambition' to push boundaries when aiming to solve new problems.		-