	Y12 Applied	CH11 7.4	Mechanics – Variable Acceleration	Lessons 5

Essential Knowledge Milestones	Teaching Points
• Understand that displacement, velocity and acceleration • may be given as functions of time • Use differentiation to solve kinematic problems • Use calculus to solve problems involving maxima and minima • Use integration to solve kinematics problems • Use calculus to derive constant acceleration formulae **Success Criteria** ❑ You understand that if acceleration of a particle is variable, then it changes with time and can be expressed as a function of time ❑ You can explain time restrictions on models by considering algebraic arguments or graphical sketches ❑ You can differentiate functions of time, understanding how that allows you to move between displacement, velocity and acceleration formulae ❑ You can solve problems using these functions ❑ You can use calculus to determine maximum and minimum values of displacement, velocity and acceleration ❑ You understand that integration is the reverse of differentiation so this also allows you to move between displacement, velocity and acceleration formulae ❑ You can use calculus to derive the suvat equations **Assumed Prior Knowledge/ Links / Interleaving** <u>GCSE (9-1) in Mathematics at Higher Tier</u> • **A11** Identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically and turning points by completing the square • **A14** Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration • **A15** Calculate or estimate gradients of graphs and area under graphs (including quadratic and non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts <u>AS Mathematics – Pure Mathematics content</u> • **7, 8** Differentiation and integration of polynomials (See Units 6 and 7 of the SoW)	• Start by stating that the *suvat* formulae from Unit 7 can only be used when acceleration is constant and the motion is in a straight line. This means the speed-time or velocity-time graphs are made up of straight lines. • Draw the graph of say, $v = 2t^2 + 2t + 1$ (for $t > 0$). This is part of a parabola where the gradient is increasing so as time passes the object is accelerating more quickly. As acceleration is not constant, the *suvat* formulae will not work for this model. • Make links (using AS Pure Mathematics calculus) to the rate of change of velocity explaining that $\frac{dv}{dt}$ = gradient = acceleration. This idea that the gradient of a velocity–time graph gives acceleration should be familiar from previous work in Unit 7 and also from GCSE (9-1) in Mathematics. • Summarise the situation by talking about, velocity as the rate of change of displacement and acceleration as the rate of change of velocity. • Express these statements in the notation of calculus: $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$. • Students will also need to relate the fact that the gradient = 0 at the max or min point to this mathematical model i.e. if $\frac{dv}{dt} = 0$, then acceleration = 0, so the particle must be at max or min velocity, as it cannot accelerate (or get any faster or slower) any more at this point in time. • Return to the graph of $v = 2t^2 + 2t + 1$ (for $t > 0$) introduced at the start of Unit 9a. • From earlier work in Unit 7 and from GCSE (9-1) in Mathematics, students should know that the area under a velocity–time graph equals the displacement. • Remind students that, from their work for Pure Mathematics, the area under a curve can be found using integration. This means that the integral of the velocity expression (with respect to time) gives the displacement. • By linking integration with the reverse of differentiation, displacement and velocity can be found by integrating expressions for velocity and acceleration respectively: ○ $\mathbf{r} = \int v\,dt$ and $v = \int a\,dt$ • (Again 's' can be used in place of '**r**' for straight line motion in this section) • Move on to explain that the constant of integration, *c* needs to be found by referring back to the problem and using some (usually initial) information about the body. For example knowing that the particle starts from *O* at rest means that when $t = 0$ (initially), $s = 0$ (at *O*) and $v = 0$ (at rest). These values can be substituted to calculate *c*. • Please note Example 3 Q3. It is incredibly important for students to sketch the graph. The common error is to always assume that the max/min is at a turning point. • This same sort of idea is really key for Example 7. Students must check that there is no change of sign (or that the area they are finding is entirely above or below the x axis) Be careful for little nuances like distance or displacement. • Link the derivation of suvat back to the graphical methods of unit 9

Potential Barriers to Access /Misconceptions	Opportunities for Reasoning/Problem Solving/Proofs
• Students who draw sketches of the situation are often more successful in reaching the correct solution, so you should continue to encourage this wherever possible. • Students often ignore or don't recognise the difference between displacement and distance and so may end up discarding negative values without considering how they should be interpreted. • Students can easily forget that if the velocity becomes negative, for example when a particle stops and changes direction, they need to split the integral to calculate distance rather than displacement.	• You could extend the calculus approach to relate double differentiation and signs of $\frac{d^2s}{dt^2}$ to indicate if it is a min or max displacement. • Students need to be able to know when to differentiate and/or integrate and how acceleration = 0 gives a maximum velocity so questions like the following are useful. • A particle moves so that it's motion is modelled by the following equation, $v = 6t(3 - t)$ m s^{-1}. • Find: **a** the times when it is at rest, **b** its maximum velocity, **c** an expression for its acceleration, **d** the total distance it travels between the times it is stationary. • **Extension:** Starting with constant *a*, students can derive the earlier equations of uniform motion. Stress constants of integration, which produce *u* in $v = u + at$ and the s_0 (*s* when $t = 0$) in $s = ut + \frac{1}{2}at^2 + s_0$.

Key Mathematical Vocabulary	Distance, displacement, velocity, speed, constant acceleration, variable acceleration, retardation, deceleration, gradient, area, differentiate, integrate, rate of change, straight-line motion, with respect to time, constant of integration, initial conditions.

Personal Development	Notes	Resources
Pupils are taught that they must 'respect' each other's opinions and well-being when working collectively in class. Pupils to learn that mathematicians have 'ambition' to push boundaries when aiming to solve new problems **Resilience** – never giving up! Building confidence across the problem solving aspects of the course. **Ambition** – living life to the full – fulfilling dreams and aspirations – linking to future career and ambition plans. **Respect** – respect for others – the 9 protected characteristics **Personal Best** – First Work – Best Work every time	• The level of calculus will be consistent with the contents of AS Pure Mathematics. • The specification states the following:– $v = \frac{d\mathbf{r}}{dt}$, $a = \frac{dv}{dt} = \frac{d^2\mathbf{r}}{dt^2}$ using '$\mathbf{r}$' to represent displacement '*s*'. **r** will become the vector notation of displacement when we later analyse 2D kinematics using the **i**, **j** system (A level Mathematics – Mechanics section, see SoW Unit 8).	• Variable Acceleration Pack