THE ACADEMY OF ST NICHOLAS	Y12 Applied	CH10 8.1, 8.2, 8.4	Mechanics – Forces and Motion	Lessons 6

Essential Knowledge Milestones

- Draw force diagrams and calculate resultant forces
- Understand and use Newton's first law
- Calculate resultant forces by adding vectors
- Understand and use Newton's second law, $F = ma$
- Apply Newton's second law to vector forces and acceleration
- Understand and use Newton's third law
- Solve problems involving connected particles

Success Criteria

- ❑ You will be able to extend your forces diagrams to include numerical forces which can then be used to calculate resultant forces.
- ❑ You know that if an object is in equilibrium then its forces are balanced
- ❑ You can add force vectors in **i-j** or column notation vectors to find the resultant force knowing that if the object is in equilibrium the resultant force will be 0**i** + 0**j**
- ❑ You can find the magnitude and bearing of a resultant force
- ❑ You know that a non zero resultant force will cause the particle to accelerate in the direction of the resultant force. Newton's second law states $F = ma$
- ❑ You can solve problems involving vector forces working on a particle using $F = ma$
- ❑ You can solve problems with connected particles knowing when it is appropriate to do so!
- ❑ You can solve problems using 'hanging' and 'table' pulleys knowing that all problems use the same modelling assumptions and what the mean!

Assumed Prior Knowledge/ Links / Interleaving

GCSE (9-1) in Mathematics at Higher Tier

- **A19** Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically; find approximate solutions using a graph
- Modelling and definitions/assumptions from the introduction in Unit 6

Teaching Points

- Relate this topic back to the different types of forces defined in Unit 6b.
- Newton said '*An object continues in state of rest or uniform motion unless acted on by an external force*.' Hence one can define a force as something which causes a body to accelerate. Explain to students that 'no force acting' means a body will either be stationary or be moving with constant velocity (i.e. acceleration = zero). This is why in outer space an object keeps moving at constant speed once pushed (there are no forces to speed it up, slow it down or stop it moving.)
- So, an object at rest or constant velocity $\Rightarrow$ no resultant force; an object changing speed or direction $\Rightarrow$ resultant force. This will lead to Newton's second law in the next section.
- Newton also stated '*When an object A exerts a force on another object B there is an equal and opposite reaction force of B on A*.' Explain that if a book is on a smooth, horizontal table, the forces acting on the book are the Weight, W (vertically down) and the normal reaction, R (always at 90° to the *surface* of contact). Assuming the table surface material is strong enough to hold the full weight of the book, the two forces balance each other and there is no resultant force. The book does not move, hence it is in equilibrium.
- Ask questions such as: If the book has a mass of 5 kg, what is its weight? Therefore, what would the magnitude of the normal reaction be to guarantee equilibrium?
- Draw different examples of force diagrams to illustrate: weight, reaction, tension (in strings), thrust (in rods), compression (in light rods, springs) etc.
- To illustrate thrust, balance a book on a ruler. In which direction is the thrust force acting?
- Introduce the **i - j** notation. The forces can be given in **i - j** form or as column vectors. Questions on equilibrium will be limited to perpendicular forces so the sum of the forces must be 0**i** + 0**j** for equilibrium.
- Newton stated, '*Where there is a force, there is an acceleration (or deviation from uniform motion) and the force is proportional to the acceleration*'. Therefore $F \propto a$, and choosing the constant to suit the motion units gives $F = ma$. (Newton's second law). This is known as the 'equation of motion'.
- Explain to students that if they sum all the effects of the forces acting, in a particular direction, this will be equal to the mass x the acceleration in that direction. This process is called resolving the forces in that direction e.g. resolving horizontally, or R(→) for short. It's usually best to resolve IN the direction of the acceleration and/or perpendicular to the direction of the acceleration.
- When resolving always take the positive direction as the direction of the acceleration and put all the forces on one side of the equation and (mass x acceleration) on the other side.
- When working on connected particles problems (such as trains or pulley systems) explain to students that they should consider the whole system as well as the separate parts. Applications to be covered are lift problems, car and caravan type questions and connected particles

	passing over a smooth pulley. Consider both pulley scenarios: a pulley with both stings hanging vertically; and a pulley at the end of a horizontal table. • In terms of connected particles I have found the spring analogy useful to help them understand the direction of tension in the string/rod. I.e. If it were a spring instead and a car and trailer accelerated the string would stretch so the force that keeps it in its usual form is acting inwards. If the car put on the breaks the string would compress hence the opposite would be true in terms of the tension. • Make it clear that treating a situation as a connected particle means that we can remove the unknown tension forces hence simplifying the problem. You can only do this though given appropriate assumptions. • Pulleys are classic exam questions. Go to science and borrow some so that students can visualise the models. A common and tricky question is dealing with the scenario of when one mass hits the floor, what happens the other (distance travelled or time for string to become taut again). The classic error here is to keep acceleration the same throughout. Make sure that students know if there is a change in the system (at this point the string is slack ie no tension) they must recalculate the acceleration. Get students to construct the solutions in two phases and ensure they understand that v in phase 1 becomes u in phase 2. • This also links to connected particle problems where the tow rope snaps. • Ensure that students can see the link between F&M and kinematics. Acceleration is the link between these two worlds.
Potential Barriers to Access /Misconceptions	**Opportunities for Reasoning/Problem Solving/Proofs**
• Students are often good at drawing force diagrams, but common errors are omitting arrowheads, incorrectly labelling (e.g. 4 kg rather than 4*g*) and missing off the normal reaction. • Students can easily be confused by the vocabulary, e.g. mixing up 'resultant' and 'reaction'. • **Pulleys**: In past exam questions, most students used an equation of motion for each particle with very few 'single equation' solutions. Students may also mistakenly take the acceleration to be equal to *g* rather than the value obtained in the question. • **2 Vehicles**: In exam questions of a car-and-trailer type, students may consider the car and trailer as a single system. Common errors when resolving are: to add a tension force (when there is no rope): to consider the weight; or to confuse the positive and negative directions. • Spend time discussing tension in strings as students find this a difficult concept	• You could extend to rough inclined planes and show all the forces balancing to provide equilibrium. Resolving forces is not in the AS course (covered in A Level Mathematics – Mechanics content, see SoW Unit 5a). • For the connected particle problems, discuss, the assumptions from Unit 6a, i.e. smooth pulley, inextensible string, same tension in the string. Extend the questions so that (for a pulley question) the particle moving down eventually hits the table and the string goes slack. This means the particle moving up continues as a 'free' particle so we now apply the equations of motion with $a = -9.8 \text{ m s}^{-2}$.

Key Mathematical Vocabulary	Force, newtons, mass, weight, gravity, tension, thrust, compression, air resistance, reaction, driving force, braking force, resultant, force diagram, equilibrium, inextensible, light, negligible, particle, smooth, uniform, pulley, string, retardation, free particle.

Personal Development	Notes	Resources
Pupils are taught that they must 'respect' each other's opinions and well-being when working collectively in class. Pupils to learn that mathematicians have 'ambition' to push boundaries when aiming to solve new problems	• Resolving forces is not in the AS course and equilibrium problems will not require forces to be resolved. Scenarios will be restricted to forces in two perpendicular directions, or simple cases of forces, given as 2D vectors.	• F&M Pack • Go to science and borrow some pulleys and equipment. Nothing matches up to actually seeing!

Resilience – never giving up! Building confidence across the problem solving aspects of the course. **Ambition** – living life to the full – fulfilling dreams and aspirations – linking to future career and ambition plans. **Respect** – respect for others – the 9 protected characteristics **Personal Best** – First Work – Best Work every time	• Resolving forces and the concept of a friction force (which opposes relative motion) is covered in A level Mathematics – Mechanics section, see SoW Unit 5.	