THE ACADEMY OF ST NICHOLAS	Y13 Applied	CH03 3.6,3.7	Statistics Normal Distribution approx. to the Binomial & Hypothesis Testing	Lessons 3

Essential Knowledge Milestones

- understand and be able to apply a continuity correction;
- be able to use the Normal distribution as an approximation to the binomial distribution.
- be able to conduct a statistical hypothesis test for the mean of the Normal distribution;
- be able to interpret the results in context.

Success Criteria

- ❑ You can use the Normal distribution as an approximation to the binomial distribution
- ❑ You can conduct a statistical hypothesis test for the mean of the Normal distribution

Assumed Prior Knowledge/ Links / Interleaving

GCSE (9–1) in Mathematics at Higher Tier

- Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically

AS Mathematics – Statistics content

- Probability calculations, independent events
- Properties of the binomial distribution
- **P**robability is the area under a curve
- Use appropriate language of statistical hypothesis testing

Teaching Points

- Begin by recapping the binomial distribution and making clear that the Normal distribution is continuous and the binomial distribution is discrete.
- Students need to understand that the binomial distribution can be approximated by the Normal distribution when n is large and p is close to 0.5. Look at the parameters needed for the Normal distribution (μ and σ^2) and cover how the mean and variance are approximated from the binomial distribution ($\mu = np$ and $\sigma^2 = np(1-p)$). Students should be confident with the notation that $X \sim B(n,p)$ is approximated by $Y \sim N(np, np(1-p))$. Encourage them to write both distributions when answering questions involving an approximation.
- When calculating probabilities for a binomial distribution which has been approximated by the Normal distribution it is important to remember that a discrete distribution has become a continuous distribution and the continuity correction needs to be introduced. It is useful here to look back at the bar chart diagrams you used in year one.
- To help students understand the continuity correction label the edges of say the 8 bar with the boundaries 7.5 and 8.5 etc. If for the binomial distribution the probability $P(X \leq 8)$ is required then shade the whole of the 8 bar and below; this indicates that the corresponding Normal probability is $P(X < 8)$.
- Using the binomial distribution, for $P(X < 8)$ the 8 bar won't be shaded but every bar below it will. This indicates that using the Normal distribution the probability will be $P(Y < 7.5)$. The same principle works for probabilities of the form $P(X > a)$ and $P(X \geq a)$. Make sure students are clear that for Normal probabilities < and ≤ are interchangeable as it is a continuous distribution.
- Once students have mastered using the Normal distribution as an approximation to the binomial distribution make sure you give them the opportunity to solve questions where they have to explain which distribution can be used before solving the problem, and whether an approximation is necessary or not. Ensure they are competent in explaining why they have chosen the distribution or approximation, clearly stating the relevant properties of their chosen distribution. They should also be able to describe why they have discounted the use of a distribution or approximation.
- Remind students of the properties of the Normal distribution and the parameters it uses. Questions could involve a known, given or assumed variance and students should be aware of this
- Hypothesis tests need to be carried out for the mean of the Normal distribution. For $X \sim N(\mu, \sigma^2)$, students need to understand that for a sample, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

Potential Barriers to Access /Misconceptions	Opportunities for Reasoning/Problem Solving/Proofs
• Correctly applying continuity corrections can prove difficult with students either not applying one or otherwise adding 0.5 rather than subtracting or vice versa. • Refer back to the formula used to translate *X* into *Z* and make sure students know they can test μ using $\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1^2)$. • This is the third type of hypothesis test that students are expected to be able to carry out so the importance of using the correct parameter in the hypotheses should be emphasised here. Hypotheses for the Normal distribution should be stated in terms of μ. • Common errors in exam situations include: not expressing hypotheses precisely enough; using an incorrect parameter or not using a parameter at all; incorrectly applying the continuity correction; and not giving a conclusion or answer to the question using the given context. • As in all cases conclusions need to be written clearly and in the context of the question	• Use an example when n is large and p is close to 0.5 and look at a variety of cumulative probabilities in both the binomial and the Normal distributions to show students how good the approximation is to the binomial distribution. • Now all types of hypothesis testing have been covered students should be given mixed problems of all types from years one and two in order to practise distinguishing between tests. Ensure all hypotheses are written in terms of the correct parameter.

Key Mathematical Vocabulary	Binomial, discrete distribution, discrete random variable, uniform, cumulative probabilities Normal, mean, variance, continuous distribution, histogram, inflection, appropriate probability distribution.

Personal Development	Notes	Resources
Pupils are taught that they must show 'compassion' when working collaboratively as peer support may be required by those potentially making mistakes.		