THE ACADEMY OF ST NICHOLAS	Y13 Pure	CH11 11.6	Integration By parts	Lessons 3

Essential Knowledge Milestones	Teaching Points
• be able to integrate an expression using integration by parts; • be able to select the correct method for integration and justify their choices. **Assumed Prior Knowledge/ Links / Interleaving** • AS: Knowledge of e^x and $\ln x$ • AS: Laws of logarithms • AS: Trigonometry • AS: Differentiation and integration **Potential Barriers to Access/Misconceptions** • Common errors when integrating by parts include: choosing *u* and dv incorrectly (in particular ln *x* must always be chosen as *u*); algebraic errors – especially if they do not remove any common factors to outside the integral sign; incorrect coefficients when integrating dv; and sign errors where sin and cos are involved. • The method of integration by parts may be specified in the question. – students don't	• It is a good idea to show how the product rule for differentiation can be integrated on both sides to derive the 'by parts' formula (which is given in the formulae booklet). • Students are usually able to start questions using this method but struggle to get to full solutions and will require lots of practice with algebraic manipulation. • Time should be spent discussing the choice of *u* and dv. It is usually advisable to select the polynomial to be the *u* as it simplifies to a lower power after calculating d*u*, thus making the second integral easier than the original question. • Students should recognise that ln *x* cannot be integrated simply and should therefore always be chosen as *u*. • ln *x* itself can be integrated using this method taking *u* = ln *x* and dv = 1 (as we cannot integrate ln *x*, but can differentiate it to give $\frac{1}{x}$). The dv becomes more complicated, but then simplifies in the second integral with the $\frac{1}{x}$. • More able students should be able to access questions where it is necessary to use integration by parts twice (e.g. $u = x^2$).

Questions & Prompts	Opportunities for Reasoning/Problem Solving/Proofs
• Is substitution a useful technique for finding $\int_0^{\frac{\pi}{2}} x\sin x\,dx$?	• I Consider the integral of $e^x \cos x$ and show that the application of 'by parts' loops back to the original question. Refer to the equation x = 4 – x and contrast this with the structure of this example. • Let the original question be *I* (for integral) and this can lead to 2*I* = … . • [This is a pre-requisite for reduction formulae in Further Pure Mathematics.] • Students should integrate functions such as $\int x(x+3)^6 dx$ using both 'by parts' and 'substitution' to show that they give the same answer. This is a good activity for discussion as initially they appear to be different, but after some algebraic manipulation give the same answer.

Key Mathematical Vocabulary	Integral, inverse, differential, coefficient, index, power, negative, reciprocal, natural logarithm, ln |x|, coefficient, exponential, identity, sin, cos, tan, sec, cosec, cot, e^x.

Personal Development	Notes	Resources
Independent work that requires a planned approach in terms of time management and self-discipline in order to meet deadlines within exam conditions.		