	Y13 Pure	CH11 11.8	Integration trapezium rule.	Lessons 1

Essential Knowledge Milestones

- be able to use the trapezium rule to find an approximation to the area under a curve;
- appreciate the trapezium rule is an approximation and realise when it gives an overestimate or underestimate.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: Area under a curve

Potential Barriers to Access/Misconceptions

- When using the trapezium rule students sometimes mix up the number of strips and the number of *x* or *y* values.
- The other main place marks are lost is not giving the final answer to three significant figures.

Opportunities for Reasoning/Problem Solving/Proofs

The following exam question shows a modelling example:

A river, running between parallel banks, is 20 m wide. The depth, *y* metres, of the river, measured at a point x metres from one bank, is given by the formula:

$y = \frac{1}{10}x\sqrt{20-x}$, $0 \le x \le 20$

(*a*) Complete the table below, giving values of *y* to 3 decimal places.

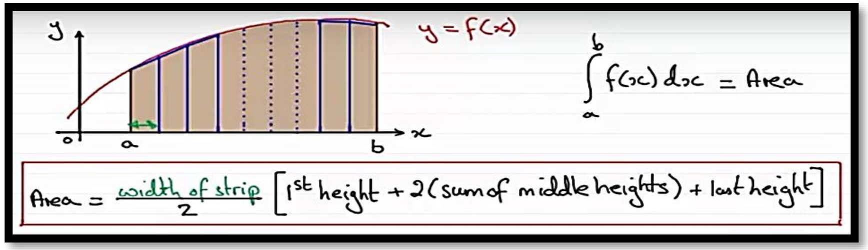
x	0	4	8	12	16	20
y	0		2.771			0

(*b*) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

Teaching Points

Make a direct link with the previous section and how to find an estimate for the area under a curve by dividing it into a finite number of strips. Sometimes an estimate is all that we need, and sometimes the integral is very complicated (or sometimes impossible) to integrate and so we have to estimate the area numerically.

The trapezium rule is given in the formula book (and may have also been covered in GCSE (9-1)). Students who struggle with algebra sometimes prefer to use the word version below:



Some students may be able to derive the rule by adding all the individual strips areas (i.e. $\frac{1}{2}h(y_0 + y_1) + \frac{1}{2}h(y_1 + y_2) + \ldots$) and then factorising to give the trapezium rule as in the formula book.

Ask students to calculate $\int_0^1 xe^{2x}$ by using integration by parts and also by completing the table and using the trapezium rule (this is the quicker method). They should compare the answers they get using the different methods.

x	0	0.2	0.4	0.6	0.8	1
y = xe^2x^	0	0.29836		1.99207		7.38906

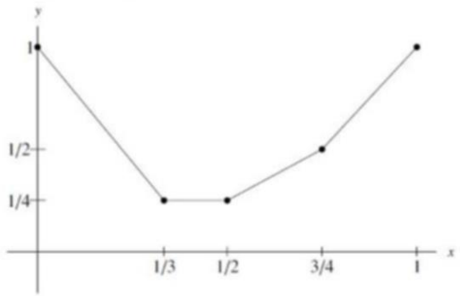
Another example of the type of question that may be asked is:

Evaluate $\int_0^1 \sqrt{2x+1}\,\mathrm{d}x$ using the values of $\sqrt{2x+1}$ at x = 0, 0.25, 0.5, 0.75 and 1.

Make a sketch of the graph to determine whether the trapezium rule gives an over-estimate or an under-estimate of the exact value of the integral.

Questions & Prompts

The graph of $y = f(x)$ is shown for $0 \le x \le 1$



The trapezium rule is then used to estimate $\int_0^1 f(x)\ dx$

by dividing $0 \le x \le 1$ into n equal intervals. The estimate will equal the actual integral when n is a multiple of what?

Key Mathematical Vocabulary	Integral, inverse, differential, coefficient, index, power, negative, reciprocal, natural logarithm, ln |x|, coefficient, exponential, identity, sin, cos, tan, sec, cosec, cot, e^x.

Personal Development	Notes	Resources
Pupils are taught that they must be able to put into 'action', from feedback provided by their teacher. They will learn to strive for their 'personal best' in the quality of their outcomes, in order to successfully meet criteria.		