	Y13 Pure	CH11 11.8	Integration Area under graphs or between two curves	Lessons 3

Essential Knowledge Milestones	Teaching Points
• understand and be able to use integration as the limit of a sum; • understand the difference between an indefinite and definite integral and why we do not need + c; • be able to integrate polynomials and other functions to find definite integrals, and use these to find the areas of regions bounded by curves and/or lines; • be able to use a definite integral to find the area under a curve and the area between two curves. • **Assumed Prior Knowledge/ Links / Interleaving** • AS: Knowledge of e^x and ln x • AS: Laws of logarithms • AS: Trigonometry • AS: Differentiation and integration • Trapezium rule **Potential Barriers to Access/Misconceptions** • The method for answering these types of exam questions is often understood, but many students loose accuracy marks due to arithmetical errors or using incorrect limits.	Begin by showing a sketch of the curve and spit the area below it into thin strips, Now each strip is of elemental width δx, so the approximate area of each strip is $y\delta x$, where *y* is the height of each strip measured on the *y*-axis. If we sum all the strips, this would give us the total area below the curve. If the first strip starts at the point (2, 0) and the last strip ends at (4, 0), these become the limits on the definite integral. We can think of '4' as the area up to 4 and '2' as the area up to 2 (both measured across from the *y*-axis or x = 0). We have seen from work on series, that we can use the sigma notation for sums so we can represent the area as $\sum y\delta x$. As δx gets thinner and thinner, the area becomes more accurate as the strips become more like rectangles. (This links nicely with the trapezium rule in the next sub-unit.) We say that 'in the limit, as δx approaches zero' the sum becomes continuous rather than discrete and we can replace *y* with f(x) and $y\delta x$ becomes f(x)δx. It happens that the rule for integration (which so far has only been used as the reverse of differentiation) gives the exact area under the curve. We can substitute in *a* and *b*, where the area's strips began and ended, as the limits of integration. The $y\delta x$ becomes f(x)δx and for the integral becomes f(x)dx. In other words the δx is the dx we have always understood as 'with respect to x'. This leads to, $\int_a^b \mathrm{f}(x)\,\mathrm{d}x = \lim_{\delta x \to 0} \sum_{x=a}^{b} \mathrm{f}(x)\,\delta x$ Do lots of work on finding areas that require more than just a simple integral to be evaluated, for example when some of the area is below the x-axis or when finding the area between a line and a curve. For example: Find the finite area bounded by the curve *y* = 6x – x^2 and the line *y* = 2x. Find the finite area bounded by the curve *y* = x^2 – 5x + 6 and the curve *y* = 4 – x^2. Encourage students to always do a sketch or use a graph drawer to help with such questions.

Questions & Prompts	Opportunities for Reasoning/Problem Solving/Proofs
• Can I have a negative area? • Include questions where the area is found between a curve and the *y*-axis using ∫x dy, with *y*-coordinates as limits. • Finally, consider areas which are bounded by curves defined by other types of functions, e.g. *y* = e^{2x} or y = ln x.	• Consider questions which have part of the graph below the *x*-axis, in which the area is negative. This time the roots are vital as we have to create two separate regions to calculate the total area. Show that just integrating between the start and end points will give a wrong result as the areas will subtract form each other. • Sometimes, you can create a new equation by subtracting the two areas *before* you integrate (when you have two curves and have to find the area between them): • $\int_a^b y_1\,\mathrm{d}x - \int_a^b y_2\,\mathrm{d}x = \int_a^b (y_1 - y_2)\,\mathrm{d}x$ • Care is needed with this method, and you should emphasise to students that they need to sketch it first making sure y_1 is higher than y_2.

Key Mathematical Vocabulary	Integral, inverse, differential, coefficient, index, power, negative, reciprocal, natural logarithm, ln $\|x\|$, coefficient, exponential, identity, sin, cos, tan, sec, cosec, cot, e^x.

Personal Development	Notes	Resources
Pupils are taught that they must 'respect' each other's opinions and well-being when working collectively in class. Pupils to learn that mathematicians have 'ambition' to push boundaries when aiming to solve new problems.		