	Y12 Pure	CH09 9.3,9.4, 9.5,9.6,9.7,9.8,	Differentiation Products, Quotients, implicit and parametric functions	Lessons 6

Essential Knowledge Milestones	Assumed Prior Knowledge/ Links / Interleaving
• be able to differentiate composite functions using the chain rule; • be able to differentiate using the product rule; • be able to differentiate using the quotient rule; • be able to differentiate parametric equations; • be able to find the gradient at a given point from parametric equations; • be able to find the equation of a tangent or normal (parametric); • be able to use implicit differentiation to differentiate an equation involving two variables; • be able to find the gradient of a curve using implicit differentiation; • be able to verify a given point is stationary (implicit).	• GCSE: Coordinate geometry • GCSE: Changing the subject of the formula, and substitution • GCSE: Graphs of linear, quadratic and trigonometric functions • AS: Coordinate geometry • AS: Trigonometric identities • AS: Differentiation • Functional notation including f′(x)

Teaching Points	Teaching Points
• Most students will be able to differentiate simple instances of e^{3x}, sin 3x and ln 3x without needing formal methods such as $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$. • Many will also be able to differentiate expressions such as $(3x + 7)^5$ without using the formal method $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1}f'(x)$. • When using the chain rule and the formula $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, initially *u* can be given to students, but they must be able to choose their own *u* and should move onto this quickly. Encourage students to lay work out carefully, using correct notation and $\frac{dy}{du}$ and $\frac{du}{dx}$, not always $\frac{dy}{dx}$. • Teaching should focus on *how* students know a function needs to be differentiated using the chain rule (or function of a function) and *why* a particular *u* is selected. • As an introduction for the product rule, ask the students to differentiate x^4. If you rewrite this as the product $(x^2)(x^2)$ and differentiate each part separately, it does not match $4x^3$. Using the product rule will give that match. • In a similar way, writing x^4 as $\frac{x^3}{x}$ can lead into the quotient rule. • Work involving the product and quotient rule often breaks down because of weak algebraic skills (PRACTICE) • Show that the product rule and the quotient rule give the same answers on functions that can be written in two ways, for example, $y = \frac{x+1}{x+2}$ and $y = (x + 1)(x + 2)^{-1}$. • Also show that the chain rule and the product rule give the same derivative for $\cos^2 x$ and $\sin^2 x$. • Use the product and quotient rules to derive the differentials of some key trigonometric expressions. For example $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$ using the quotient rule giving $\sec^2 x$.	• For parametric differentiation, make links with the chain rule to give $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ • Stress that we often substitute in the value of the parameter *t* at the point which we need to find the gradient • Many questions will involve trigonometric functions, so students must be fluent at differentiating these. • For implicit differentiation, consider the equation of a circle, $x^2 + y^2 = 16$. To differentiate this function we would have to make *y* the subject of the formula. Sometimes this can be difficult or even impossible. • Make sure students can confidently differentiate terms like x^2y using implicit differentiation. Finally, stress that we need to substitute in *both x* and *y* coordinates to find the gradient at a certain point. • Students may have to apply the product or quotient rules in implicit differentiation questions and should be given examples of this. In exam questions students are almost always required to find the gradient through implicit differentiation. • Take a point on a circle or another type of curve and find the gradient using two both parametric and implicit differentiation. Then find the equation of tangent and/or normal and see that both methods give the same answer.

Potential Barriers to Access/Misconceptions	Opportunities for Reasoning/Problem Solving/Proofs
• Common errors involve: not using the method specified; algebraic errors when manipulating expressions; and being unable to identify the need of the product rule and instead simply differentiating the separate parts and multiplying. • Failing to be clear which variable is being differentiated and with respect to which other variable and generally using $\frac{dy}{dx}$ as a universal notation for "the differential coefficient of" • Assuming a more simple model for product and chain rule (e.g. just the product of differentials) • Difficulty recognising products and composite functions	• Although repeated chain rule questions rarely appear in the exam they provide good extension material and provide an excellent test of good method and correct mathematical notation. Extend the students further by asking them to look up a proof or derivation of the product and quotient rules. • Use the methods above to work out the derivative of a general exponential function, i.e. $\frac{\mathrm{d}}{\mathrm{d}x}(a^{kx}) = ka^{kx}\ln a$ • Give the students lots of mixed questions which will enable them to select the correct method. Discussion should focus on why they have selected a particular method and quick ways of identifying the correct method. • Students must be able to use all methods as a particular method is sometime specified in the exam. • Some questions require a trigonometric identity in order to simplify the solution. • The specification states that 'differentiation of arcsinx, arccosx, and arctanx are required'. • Cover questions involving finding tangents, turning points and normals • Using the product rule on $u = yv$, prove the quotient rule formula for $\frac{d}{dx}\left(\frac{u}{v}\right)$
Questions & Prompts	
• Make up one question in which you need to use both the product rule and the chain rule • Why is the chain rule sometimes called the function of a function rule? • Tell me two ways of finding the gradient of the tangent to the circle $x^2 + y^2 = 5^2$ at the point $(3,4)$.	

Key Mathematical Vocabulary	Derivative, tangent, normal, turning point, stationary point, maximum, minimum, inflexion, parametric, implicit, differential equation, rate of change, product, quotient, first derivative, second derivative, increasing function, decreasing function.

Personal Development	Notes	Resources

Independence & resolve need to be nurtured. Execute practical unaided after being shown a demonstration of how to approach problem solving. Prepare them to work through a problem independently in life.		