	Y12 Pure	CH09 9.1,9.2,	Differentiation sin x, cos x, exponentials and logarithms	Lessons 4

Essential Knowledge Milestones

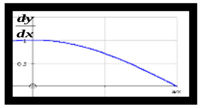
- be able to find the derivative of sin x and cos x from first principles.
- be able to differentiate functions involving e^x, ln x and related functions such as $6e^{4x}$ and 5 ln 3x and sketch the graphs of these functions;
- be able to differentiate to find equations of tangents and normals to the curve.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: Coordinate geometry
- GCSE: Changing the subject of the formula, and substitution
- GCSE: Graphs of linear, quadratic and trigonometric functions
- AS: Coordinate geometry
- AS: Trigonometric identities
- AS: Differentiation
- Functional notation including f′(x)
- Functions: the chain rule is used to differentiate composite functions

Teaching Points

- Review how to differentiate polynomials from first principles.
- Sketch *y* = sin x and consider the gradient at key points by looking at slopes of tangents. If we plot the gradients then we get a shape which looks like the start of a cos graph:



- Approach the differentiation from first principles
- It is vital that students understand the functions e^x and ln x and do not just learn how to differentiate them. Use a graphing tool to show that the gradient of a special curve $y = a^x$ has a gradient which is exactly a^x. In other words its rate of growth is exactly the same as its value at that point.

Potential Barriers to Access /Misconceptions

- Students often miss out minus signs or add an extra x into the answer when differentiating expressions like $e^{\frac{1}{4}x}$.
- Some students mix up $\frac{dx}{dy}$ and $\frac{dy}{dx}$ and others struggle to differentiate functions involving ln. For example given when differentiating *y* = ln 6*x* they write $\frac{1}{6x}$ rather than $\frac{1}{x}$.
- Errors with signs in $\frac{d}{dx}(sin\,x) = cos\,x$ and $\frac{d}{dx}(cos\,x) = -sin\,x$

Opportunities for Reasoning/Problem Solving/Proofs

- Ask the students to experiment with a graph-drawing package to verify that the gradient functions of sin x and cos x match the result found using first principles.
- Students must understand that the differentiation of sin x and cos x can only be used when x is in radians and that they must use radians whether stated in the question or not.
- Find gradients and normals for exponential and log functions, using graphs to check and enhance the solutions.

Questions & Prompts

- Give me an example of a number that is equal to $3\sqrt{2}$ …and another….and another
- Change one number in $(2 + \sqrt{8})(4 - \sqrt{2})$ so that the product is a rational number.
- $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$. Always true, sometimes true, never true?
- Give me an example of a number between $5\sqrt{6}$ and $6\sqrt{5}$.

Key Mathematical Vocabulary	Derivative, tangent, normal, turning point, stationary point, maximum, minimum, inflexion, parametric, implicit, differential equation, rate of change, product, quotient, first derivative, second derivative, increasing function, decreasing function.

Personal Development	Notes	Resources
Independent work that requires a planned approach in terms of time management and self-discipline in order to meet deadlines within exam conditions.		