	Y12 Pure	CH09 9.9,9.10	Differentiation Second Derivatives & Rates of change problems	Lessons 3

Essential Knowledge Milestones

- be able to find and identify the nature of stationary points and understand rates of change of gradient.
- be able to use a model to find the value after a given time;
- be able to set up and use logarithms to solve an equation for an exponential growth or decay problem;
- be able to use logarithms to find the base of an exponential;
- know how to model the growth or decay of 2D and 3D objects using connected rates of change;
- be able to set up a differential equation using given information which may include direct proportion.

Assumed Prior Knowledge/ Links / Interleaving

- A- Level: Differentiation
- A-Level: Logarithms

Potential Barriers to Access/Misconceptions

- Students should be encouraged to state " $\frac{dx}{dy}$ = … when x = …", especially when finding a given answer.
- An easy mistake student may make is to mix up maxima and minima.
- Most students are able to substitute correctly into a formula for exponential growth and decay.
- When required to set up an inequality most students showed that they understood the information given and wrote down a correct opening expression, although there was uncertainty over which way the inequality
- Most students are able to substitute correctly into a formula for exponential growth and decay.
- When required to set up an inequality most students showed that they understood the information given and wrote down a correct opening expression, although there was uncertainty over which way the inequality should go. Some then simplified and solved using logarithms efficiently to get the correct answer. Some resorted to trial and improvement which was accepted for full marks if done correctly, but was worth no marks otherwise.
- When solving equations involving exponentials, knowledge of using logarithms varied widely. Many were unable to deal properly with the coefficient and the exponential term and wrote down equations in which t actually should have cancelled out.
- Some care needs to be taken when interpreting the answers to exponential growth and decay questions to ensure they are given in the correct form e.g. to the nearest year, second etc.

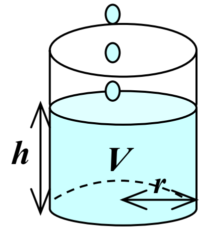
Teaching Points

- The specification states 'Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection' and 'know that at an inflection point f''(x) changes sign.'

The basic principle is usually

	$\frac{dy}{dx}$ or f'(x)	$\frac{d^2y}{dx^2}$ or f''(x)
maximum	= 0	< 0
minimum	= 0	> 0

- However show examples of curves in which $\frac{d^2y}{dx^2}$ or f''(x) = 0, where there could beva point of inflexion (or not). i.e. The rate of change of gradient is zero.
- We would need to work out f'(x) and scrutinise gradient either side of the point x. There may be positive or negative inflexion or neither (depending on the nature of the curve, which could be convex or concave).
- Use graph drawing packages to investigate the shapes and turning points of various curves of the type y = axn (n > 2)
- Modelling - exam technique is required when setting out the answers for questions, ensure the students breakdown the questions, and can recognise the key components

Questions & Prompts	**Opportunities for Reasoning/Problem Solving/Proofs**
• Typical exam questions that consolidate skills A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation $\frac{dP}{dt} = \frac{1}{22}P(11 - 2P), \quad t \geq 0$ where P, in thousands, is the population of meerkats and t is the time measured in years since the study began. Given that there are 1000 meerkats on the nature reserve when the study began, (a) determine the time taken, in years, for this population of meerkats to double, (b) show that the population cannot exceed 5500.	Consider water entering this cylinder. To work out the rate at which the height is increasing we need to calculate $\frac{dh}{dt}$. In exam questions, the rate that the volume of water increases at is often given as $\frac{dV}{dt}$. Therefore, we need to use the chain rule to create $\frac{dh}{dt}$ from $\frac{dV}{dt}$. $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ so we need a formula connecting h and V. $V = \pi r^2 h$ and from this we can work out $\frac{dV}{dh}$ and then $\frac{dh}{dV}$ etc 

Key Mathematical Vocabulary	Derivative, tangent, normal, turning point, stationary point, maximum, minimum, inflexion, parametric, implicit, differential equation, rate of change, product, quotient, first derivative, second derivative, increasing function, decreasing function.

Personal Development	**Notes**	**Resources**
Independence & resolve need to be nurtured. Execute unaided after being shown a demonstration of how to approach problem solving. Prepare them to work through a problem independently in life.		