THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH08 8.1,8.2,8.3,8.4, 8.5,8.6	Parametric Equations	Lessons 4

Essential Knowledge Milestones	Teaching Points
• understand the difference between the Cartesian and parametric system of expressing coordinates; • be able to convert between parametric and Cartesian forms. • be able to plot and sketch curves given in parametric form; • recognise some standard curves in parametric form and how they can be used for modelling. **Assumed Prior Knowledge/ Links / Interleaving** • GCSE: Coordinate geometry • GCSE: Changing the subject of the formula, and substitution • GCSE: Graphs of linear, quadratic and trigonometric functions • AS Coordinate geometry • AS Trigonometric identities • Knowledge of a variety of functions involving powers, roots, trigonometric functions, exponentials and logarithms • Differentiation: finding tangents to curves defined parametrically involves calculus techniques • Trigonometry: parametric equations often involve trigonometric functions • Functions: in the Functions unit, all functions had domain and range as subsets of the real numbers but here the range is usually 2D space; for example, $f(t) = (\sin t, \cos 2t)$ • Projectiles: the path of a projectile is modelled using parametric equations **Potential Barriers to Access/Misconceptions** • Inability to eliminate the parameter from parametric equations due to not being fluent in the use of trigonometric identities. • Not being able to simplify dy/dx once found (usually due to inefficient use of algebra techniques) • Realising that the independent variable must use radian measure instead of degrees when finding relevant coordinates. • The examiner comments for these questions illustrate how difficult students find this topic: • The final part proved very demanding and only a minority of students were able to use one of the trigonometric forms of Pythagoras to eliminate t and manipulate the resulting equation to obtain an answer in the required form. • Few even attempted the domain and the fully correct answer $0° \leq t \leq 2\pi$, was very rarely seen.	• Begin by explaining the difference between the Cartesian system, when a graph is described using $y = f(x)$, and the parametric system, which uses $x = f(t)$ and $y = g(t)$ for some parameter t. • Illustrate this by asking the class to consider $x = 5t$ and $y = 3t^2$ and to try to eliminate t from the two equations. This will give $y = \frac{3}{25}x^2$ or $25y = 3x^2$. (This is a quadratic equation – parabola.) • Repeat for $x = 5t$ and $y = \frac{5}{t}$. This becomes $y = \frac{25}{x}$ (a hyperbola). • Sometimes we need to eliminate the parameter, t, by using identities rather than substitution. • Consider $x = 3\cos t$ and $y = 3\sin t$. Squaring both equations and adding means we can use $\cos^2 t + \sin^2 t = 1$ to give $x^2 + y^2 = 9$. (This is a circle, centre (0, 0) of radius 3.) • Ask students to use similar methods to show that $x = 2 + 5\cos t, y = -4 + 5\sin t$ describes a circle centre (2, −4) with radius 5. • How do we convert from Cartesian to parametric? (We need to be in radians) For example, what are the pair of parametric equations for a circle, centre (3, 5) radius 10? • It is often easier to match the properties of a curve in parametric form than it is in its Cartesian form. • In order to establish the shapes of some well-known curves such as circles, ellipses etc., ask the students to plot the pair of parametric equations in the form of a table of values. • When plotting $x = 4\cos t$, $y = 4\sin t$ what will the range of t be? (Remember to use radians.) • Now plot $x = 4\cos t$, $y = 2\sin t$. (This is the shape mentioned in the reasoning/problem solving section of sub-unit 7a.) • What values of t will we need for $x = 5t$, $y = \frac{5}{t}$? • Investigate parametric equations which give closed loops. These will be integrated later in course to find the area of a loop, so we need to establish how values of t link plotting (direction vital). • The specification states 'Students should pay particular attention to the domain of the parameter t, as a specific section of a curve may be described.'

Questions & Prompts	Opportunities for Reasoning/Problem Solving/Proofs
• What's the same and what's different about the curve with cartesian equation $y = 2x^2 - 1$ and the curve with parametric equations $x = cos\, t\,, y = cos\, 2\, t$? • Describe which features of the parametric equations $x = 1 - t^2, y = t^3$ make it non-differentiable at the point corresponding to $t = 0$. • Give me an example of parametric equations of a curve which has a vertical asymptote.	• For the curve with parametric equations $x = 5\, cos\, t + cos\, 5\, t,\quad y = 5\, sin\, t - sin\, 5\, t$ prove that if the point with coordinates (p, q) is on the curve then so is the point with coordinates (q, p). What does this tell you about the curve? • What shape is given by x = 4 cos *t*, *y* = 2 sin *t*? • Name and properties of curve? • The trigonometric identities (such as $\sec^2 x = 1 + \tan^2 x$) can be used to convert from parametric to Cartesian form. • A shape may be modelled using parametric equations (e.g. an object moves with constant velocity from (1, 8) at *t* = 0 to (6, 20) at *t* = 5), or students may be asked to find parametric equations for a motion.

Key Mathematical Vocabulary	Parametric, Cartesian, convert, parameter *t*, identity, eliminate, substitute, circle, hyperbola, parabola, ellipse, domain, modelling.

Personal Development	Notes	Resources
Pupils are taught to be able to identify a situation whereby a particular maths skill is applied to a problem solve a question, and to have belief in their own ability.		