THE ACADEMY OF ST NICHOLAS	**Y12 Pure**	**CH07** 7.1,7.2,7.3,7.4,7.6	**Trigonometry** **Addition Rules & Trigonometric Identities**	**Lessons** **4**

Essential Knowledge Milestones

- be able to prove geometrically the following compound angle formulae for sin ($A \pm B$), cos ($A \pm B$) and tan ($A \pm B$);
- be able to use compound angle identities to rearrange expressions or prove other identities;
- be able to use compound angle identities to rearrange equations into a different form and then solve;
- be able to recall or work out double angle identities;
- be able to use double angle identities to rearrange expressions or prove other identities;
- be able to use double angle identities to rearrange equations into a different form and then solve.
- be able to construct proofs involving trigonometric functions and previously learnt identities.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: basic trigonometric ratios
- AS: Trigonometry
- Calculus: use of trigonometric identities for integration
- Transformation of graphs: $y = \sin x \cos x$ is a transformation of $y = \sin x$ (since it is the same as $y = \frac{1}{2}\sin 2x$)

Teaching Points

- A good introduction is to ask the class to work out sin (30 + 60)°.It is equal to sin (90)° = 1. Go on to ask whether sin 30° + sin 60° gives the same value (either using a calculator or using surds). They should discover that the values are different. Explain that the reason for this is that you can't simply multiply out functions in this way.
- This leads in to explaining why compound angle formulae are needed to calculate sin ($A + B$).
- Unit 1 above gives an example of a geometric proof by deduction for sin ($A + B$).
- Care needs to be taken when using the result to extend to sin ($A - B$) for negative values. Students will need to remember that cos ($-B$) = cos B and that sin ($-B$) = $-$sin (B).
- Extend these formulae by substituting $A = B$ to derive the double angle formulae
- Show that there is only one version of $\sin 2x = 2\sin x\cos x$, but the basic version of $\cos 2x = \cos^2 x - \sin^2 x$, can be re-written by substituting $\cos^2 x + \sin^2 x = 1$ (from AS Mathematics – Pure Mathematics) into two different versions (exclusively in sin x or cos x).
- A critical part of future questions and proofs involves choosing the correct version of the compound and/or double angle formulae.
- Proving trigonometric identities is something that challenges many students and is considered by some to be the most challenging part of the course.
- The basic principles are the same as in Unit 1 (Proof): manipulate the LHS and use logical steps to make it to match the RHS or vice-versa. (Sometimes both sides can be manipulated to reach the same expression.) Make sure you explain why we use ≡ rather than = (PRACTICE Needed).
- Ensure when 'Hence', seen students should be encouraged to use the result in part of the (a) above
- Students now need to explain fully that $-1 \le \cos 2\theta \le 1$, and so $\cos 2\theta = 2$ has no solutions.

Potential Barriers to Access /Misconceptions

- The most common errors are sign errors when using the compound and double angle formulae.
- These questions often prove to be the most demanding on the paper and serve to differentiate between students.
- Students need to make sure they include all steps in the proof with full explanation.
- Not knowing the formulae: an over-reliance on the formulae booklet and so not appreciating that, for example, $\sin x \cos x$ can be written in a more helpful form.
- Mis-use of the principle angle e.g. $\arcsin\left(\frac{-2}{3}\right) \approx -41.8°$ then not proceeding to give the angles in the correct range.

Opportunities for Reasoning/Problem Solving/Proofs

- Derive and cover examples using half angle formulae by adapting the double angle versions.
- The next sub- unit will look at how to solve equations of the type $a\cos\theta + b\sin\theta = C$, using compound angles to rewrite and simplify the expression on the left hand side.
- The specification says 'Students need to prove identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.'

	Questions & Prompts
	• Find two ways of solving the equation $sin\,2\,\theta = sin\,\theta$ • Explain connections between the graph of $y = cos^2 x$ and the graph of $y = cos\,2\,x$. • Prove $sin(A + B) = sin\,A\,cos\,B + cos\,A\,sin\,B$ from a diagram • $A, B$and C are the angles of a non right-angled triangle. Prove that $tan\,A + tan\,B + tan\,C = tan\,A\,tan\,B\,tan\,C$.

Key Mathematical Vocabulary	sine, cosine, tangent, secant, cosecant, cotangent, SOHCAHTOA, exact, symmetry, periodicity, identity, equation, interval, quadrant, degree, radian, asymptote, approximation, identity, proof.

Personal Development	Notes	Resources
Pupils are taught to be honest and 'truthful' in the judgments they make when they self-assess their learning as it serves to aid the teacher in planning for future learning or revisiting material for overlearning & embedding.		