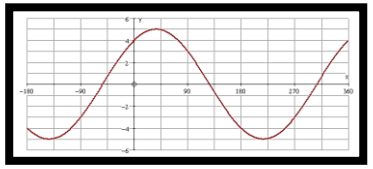
THE ACADEMY OF ST NICHOLAS	Y12 Pure	CH07 7.5,7.7	Trigonometry R Rules & Modelling	Lessons 4

Essential Knowledge Milestones	Teaching Points
• be able to express $a\cos\theta + b\sin\theta$ as a single sine or cosine function; • be able to solve equations of the form $a\cos\theta + b\sin\theta = c$ in a given interval. • be able to use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	• Start by drawing a graph of, say, 4 cos x + 3 sin x to show that is has the basic sin–cos shape. Where are the coordinates of the maximum or minimum points? It approximately fits 5 cos (x – 40°). •  • Equating 4 cos x + 3 sin x to an expanded form of *R* cos (x – *a*) gives: o 4 cos x + 3 sin x ≡ *R* cos x cos *a* + *R* sin x sin *a* • Equating coefficients leads to: o *R* sin *a* = 3 and *R* cos *a* = 4. • By squaring and adding we obtain *R* = 5, and by dividing we obtain *a* = 36.9°. (This confirms the approximate fit above.) • Move on to solving equations of the type *a* cos θ + *b* sin θ = *c* using *R* cos (x ± *a*) or *R* sin (x ± *a*) as the first step. Effectively, the question reduces to a trigonometry equation like those done in Pure Paper 1, but at this level the angles could be in radians. • Links can be made with simple harmonic motion in further mechanics, where a sin and/or cos curve could model the height of the tide against a harbour wall. When is it safe for the ship to come into the port? • For kinematics the velocity equation could be expressed as *v* = 3sin (2*t*) m s^{-1}. The times at which the object is stationary or at maximum speed could be analysed (no calculus at this stage). • An oscillating share price could be modelled using trigonometric equations. Ask students: when is the best time to buy and sell?
Assumed Prior Knowledge/ Links / Interleaving	
• Double angle identities to rearrange expressions or prove other identities; • Double angle identities to rearrange equations into a different form and then solve.	
Potential Barriers to Access/Misconceptions	
• Examiner comments suggest that the part of the calculation which causes most problems is working out the angle *a*: • When writing *a* cos θ + *b* sin θ into the form *R* sin(θ – *a*) most students found the value of *R* correctly, the same was not true of the angle *a*. Some students seemingly failed to notice that a was given as an acute angle. • When solving an equation of the form *a* cos θ + *b* sin θ = *c* many students seemingly could not cope with the result of –39.23° that their calculator gave them and could not get the first solution. In addition, some students found the third quadrant solution only, whereas some found more than two solutions. However many students did give a fully correct solution, often by using a sketch graph to help them decide where the solutions lay.	
Questions & Prompts	**Opportunities for Reasoning/Problem Solving/Proofs**

• Problems could involve (for example) wave motion, the height of a point on a vertical circular wheel, or the hours of sunlight throughout the year. Angles may be measured in degrees or in radians.'	• Ask students whether they can relate the R and a to the basic properties of the curve. • Think about the maximum/minimum value and where it occurs. • $h = a\cos t + b\sin t$ will model the tide height, h, and makes a good link with the previous section. ($t \geq 0$) Is t in degrees or radians?

Key Mathematical Vocabulary	identity, equation, interval, quadrant, degree, radian,

Personal Development	**Notes**	**Resources**
Independent work that requires a planned approach in terms of time management and self-discipline in order to meet deadlines within exam conditions.		