THE ACADEMY OF ST NICHOLAS	**Y12 Pure**	**CH06** 6.1,6.2,6.3,6.4, 6.5,	**Trigonometric Functions** **Sec, Cosec, Cot definitions, identities and graphs**	**Lessons** **4**

Essential Knowledge Milestones

- understand the secant, cosecant and cotangent functions, and their relationships to sine, cosine and tangent;
- be able to sketch the graphs of secant, cosecant and cotangent;
- be able to simplify expressions and solve involving sec, cosec and cot;
- be able to solve identities involving sec, cosec and cot;
- know and be able to use the identities $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \mathrm{cosec}^2 x$ to prove other identities and solve equations in degrees and/or radians
- be able to work with the inverse trig functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$;
- be able to sketch the graphs of $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: basic trigonometric ratios
- AS: Trigonometry
- Calculus: use of trigonometric identities for integration

Potential Barriers to Access/Misconceptions

- The most common errors in these questions involve using wrong notation, for example $\sin x^2$ instead of $\sin^2 x$, or making algebraic mistakes. Students sometimes struggle to deal with more complicated functions such as cosec $(3x + 1)$ and do not always recognise where trigonometric identities can be used.
- Thinking of inverse trigonometric functions as reciprocal trigonometric functions
- Using incorrect formulae; e.g. using $cos\,\theta = 1 - sin\,\theta$ instead of $cos^2\,\theta = 1 - sin^2\,\theta$
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Teaching Points

- Introduce students to the reciprocal trigonometric functions secant θ, cosecant θ and cotangent θ.
- A good way to introduce these as reciprocal trig functions is to start by asking whether there is another way of writing x^{-1}. This should lead to the answer $\frac{1}{x}$. If we try this with $\sin^{-1}\theta$ it is not the same meaning as $\frac{1}{\sin\theta}$, so we need to name a different function cosec θ. (Contrast this with inverse trig functions looked at later in this section)
- To help students remember which reciprocal function goes with sin, cos and tan, point out that the **third** letter of these new functions, gives the name of the trig function in the denominator, i.e.
- se**c** $\theta = \frac{1}{\cos\theta}$ co**s**ec $\theta = \frac{1}{\sin\theta}$ co**t** $\theta = \frac{1}{\tan\theta}$
- You should also point out that cot θ can be written as the reciprocal of tan θ to give $\frac{\cos\theta}{\sin\theta}$.
- Students will be expected to know what the graphs of each of the reciprocal and inverse functions look like and their key features, including domains and ranges. The relationships between the graphs and their originals can be explored on graphical calculators or graphing Apps.
- Show students how to work out new trigonometric identities by dividing $\sin^2\theta + \cos^2\theta = 1$ (from AS Mathematics – Pure Mathematics) by $\cos^2\theta$ or by $\sin^2\theta$ to give the two new identities: $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \mathrm{cosec}^2\theta$.
- This is a good alternative to simply remembering the identities and lessens the chance of mixing them up.
- It is a good idea to use the new identities to solve trigonometric equations (which are often quadratic look-a-likes) before proving identities. Sub-unit 6f covers proving identities when all the available formulae have been covered.

Questions & Prompts

Opportunities for Reasoning/Problem Solving/Proofs

• Give me two examples of trig functions with asymptotes at $x = \frac{\pi}{3}$. How would you explain why $tan\,89.9° \approx 10 \times tan\,89°$?	• To contrast **reciprocal** trig functions students will also need to be familiar with the **inverse** functions of $\sin\theta$, $\cos\theta$ and $\tan\theta$. They will again need an understanding of the graphs of $\arcsin\theta$, $\arccos\theta$ and $\arctan\theta$. Refer back to the work on functions and emphasise that for arcsin, arccos and arctan to be true functions there must be a one-one relationship between domain and range and so the domains must be restricted to $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. • Prove the trig identities $sec^2\,\theta = 1 + tan^2\,\theta$ and $cosec^2\,\theta = 1 + cot^2\,\theta$ using Pythagoras's Theorem

Key Mathematical Vocabulary	tangent, secant, cosecant, cotangent, identity, equation, interval, quadrant, degree, radian, asymptote, identity, proof.

Personal Development	**Notes**	**Resources**
Pupils are taught that they must show 'resilience' in their approach to completing questions always showing their working to ensure they achieve method marks.		