THE ACADEMY OF ST NICHOLAS	Y13 Pure	CH05 5.1,5.2,5.3,5.4, 5.5	Radians: Arc, Sectors & Exact values, Small Angles	Lessons 4

Essential Knowledge Milestones	Teaching Points
• understand the definition of a radian and be able to convert between radians and degrees; • know and be able to use exact values of sin, cos and tan; • be able to derive and use the formulae for arc length and area of sector. • Understand and use the standard small angle approximations of sine, cosine and tangent i.e. $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$ where θ is in radians **Assumed Prior Knowledge/ Links / Interleaving** • Sine and cosine function • Length of arc and area of sector **Potential Barriers to Access/Misconceptions** • A common exam mistake is for students to have their calculators set in the wrong mode resulting in the loss of accuracy marks. • Students may try to use these approximations when angles are measured in degrees rather than radians. • Mixing up sector and segments	• Ensure all students know how to change between radian and degree mode on their own calculators and emphasise the need to check which mode it is in. • Radian measure will be new to students and it is important that they understand what 1 radian actually is. • Make sure students know that 'exact value' implies an answer must be given in surd form or as a multiple of π. They need to know the exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ (and their multiples) and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ (and their multiples). • Emphasise the need to always put a scale on both axes when drawing trigonometric graphs; students must be able to do this in radians. • Make links between writing the trig ratio of any angle (obtuse/reflex/negative) to the trig ratio of an acute angle and to the trig graphs. (Do not rely on the CAST method as this tends to show a lack of understanding.) • Derive the formulae for arc length and area of a sector by replacing the $\frac{\theta}{360°}$ in the GCSE formulae with $\frac{\theta}{2\pi}$. The πs cancel giving length of arc = rθ and area of sector = $\frac{1}{2}$r2θ. • Cover examples which will involve finding the area of a segment by subtracting a triangle from a sector. • Specification states:- $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$ where θ is in radians • Experiment with trigonometric graphs and a graph-drawing package by reading off values near the origin and zooming into small angles so the students get a feeling for this new concept. • The formal proof is based on considering the area of a sector in which the angle is so small, the shape becomes a right-angled triangle (since the curved part is straightened). • By considering the area of the triangle within the sector, the area of the sector and the area of the right angled triangle we can see that • The small angle approximations can be used to give estimated values of trigonometric expressions. For example, $\frac{\cos 3x - 1}{x \sin 4x}$ approximates to $-\frac{9}{8}$ (when x is small)
Questions & Prompts	**Opportunities for Reasoning/Problem Solving/Proofs**
• These approximations only work when the small angles are measured in radians. Why don't the approximations work in degrees?	• One radian can be defined as 'the angle at the centre of a circle which measures out exactly one radius around the circumference.' Therefore, using $C = 2\pi r$, we can conclude that the full circumference, C is made up of 2π radians. This means 360 is equivalent to 2π radians.

Key Mathematical Vocabulary	Pythagoras, trigonometry, sine, cosine, tangent, exact, symmetry, equation, degree, radian, circular measure, small angles, approximation, identity, proof.

Personal Development	**Notes**	**Resources**
Learn to accept that initial answers will require modification or additional notes to ensure that method marks are picked up and exam technique is improved		