	Y13 Pure	CH03 3.5,3.6,3.7	Sequences & Series Sigma Notation, Recurrence & iterations	Lessons 4

Essential Knowledge Milestones

- be familiar with $\sum$ notation and how it can be used to generate a sequence and series;
- know how this notation will lead to an AP or GP and its sum;
- Know that $\sum_1^n 1 = n$.
- know that a sequence can be generated using a formula for the *n*th term or a recurrence relation of the form $x_{n+1} = f(x_n)$;
- know the difference between increasing, decreasing and periodic sequences;
- understand how a recurrence relation of the form $U_n = f(U_{n-1})$ can generate a sequence;
- be able to describe increasing, decreasing and periodic sequences.

Teaching Points

- The key to understanding the concept of $\sum$ is to look at the limit values and substitute them into the *n*th term formula to generate the terms of the sequence.
- Emphasise to students that they must take care when finding the starting point and never assume it starts with $n = 1$.
- Students may initially find the $\sum$ notation tricky, particularly if they are not asked to find the sum of first *n* terms, but instead asked to find, e.g. the 7th to the 20th.
- Work with sequences including those given by a formula for the *n*th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$ and link this with the work done on iterations in GCSE (9-1) Mathematics.
- Explore $x_{n+1} = f(x_n)$ type series using graphics calculators or spreadsheets. (You can draw links between this work and Unit 9 – Numerical methods.)
- Move on to general recurrence relations of the form $U_n = f(U_{n-1})$ and investigate which sequences are increasing, decreasing and periodic. Spend some time looking at the different forms of notation for recurrence relations, making sure you cover examples of increasing, decreasing and periodic sequences.

  For example:

  $u_n = \frac{1}{3n+1}$ describes a decreasing sequence as $u_{n+1} < u_n$ for all integers n

  $u_n = 2^n$ is an increasing sequence as $u_{n+1} > u_n$ for all integers n

  $u_{n+1} = \frac{1}{u_n}$ for $n > 1$ and $u_1 = 3$ describes a periodic sequence of order 2.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: Knowledge of the nth term of a sequence, although in arithmetic sequences at GCSE this is unlikely to involve the use of $n - 1$ additions of the common difference.
- Logarithms and exponentials: logs can be used to solve some equations related to geometric sequences.

Potential Barriers to Access /Misconceptions

- A fairly common error is to mix up the formulae for sums and terms, for example finding S_n rather than U_n and vice-versa.
- When asked to find the limit of u_n some candidates use the sum to infinity of a geometric series.

Questions & Prompts

- What condition would lead this sequence to converge $\sum_{k=0}^{\infty} ar^k$?
- A sequence is generated by the recurrence relationship $2u_{n+1} = ku_n + 7$, what is the largest range of k for which the sequence will converge?
- A sequence is defined by the recurrence relationship $u_{n+1} = \frac{1}{3}u_n - 7$ and $u_0 = -2$. What is the limit of the sequence as $n \to \infty$?

Opportunities for Reasoning/Problem Solving/Proofs

- Challenge students to try to work out whether a sequence is an AP, GP or neither from just looking at the structure of the sigma version of a series?
- Ask students to write a series in sigma notation
- Show that $\Sigma n = \frac{1}{2}n(n + 1)$ is the sum of *n* natural numbers and relate this to the sum formula derived in the previous section.
- Think about what to do if the upper limit is infinity.
- Cover questions in which sequences can be used to model a variety of different situations. For example, finance, growth models, decay, periodic (tide height for example) etc.
- Can you tell from the structure of a recurrence relation how it will behave, and the type of sequence it will generate?

Key Mathematical Vocabulary	Sequence, series, finite, infinite, summation notation, Σ(sigma), periodicity, convergent, divergent, natural numbers, arithmetic series, arithmetic progression (AP), common difference, geometric series, geometric progression (GP), common ratio, *n*th term, sum to *n* terms, sum to infinity (S_∞), limit.

Personal Development	**Notes**	**Resources**
Pupils are taught that they must show 'compassion' when working collaboratively as peer support may be required by those potentially making mistakes.		