THE ACADEMY OF ST NICHOLAS	Y13 Pure	CH03 3.1,3.2,3.3,3.4, 3.5,3.5	Sequences & Series Arithmetic and Geometric Progressions	Lessons 3

Essential Knowledge Milestones	Teaching Points
• know what a sequence of numbers is and the meaning of finite and infinite sequences; • know what a series is; • know the difference between convergent and divergent sequences; • know what is meant by arithmetic series and sequences; • be able to use the standard formulae associated with arithmetic series and sequences; • know what is meant by geometric series and sequences; • be able to use the standard formulae associated with geometric series and sequences; • know the condition for a geometric series to be convergent and be able to find its sum to infinity; • be able to solve problems involving arithmetic and geometric series and sequences; • know the proofs and derivations of the sum formulae (for both AP and GP). **Assumed Prior Knowledge/ Links / Interleaving** • GCSE: Knowledge of the nth term of a sequence, although in arithmetic sequences at GCSE this is unlikely to involve the use of $n-1$ additions of the common difference. • Logarithms and exponentials: logs can be used to solve some equations related to geometric sequences.	• Start by recapping the work students did on sequences at GCSE (9-1) Mathematics before moving on to the new A level content, paving the way for the sigma notation in the following section. • Use practical situations, for example involving money, to illustrate APs and GPs and contrast the different ways they grow. • Find the *n*th term of a given arithmetic sequences and also use the rule to find the next two terms. • The Gauss problem (1 + 2 + … + 1000) is a good numerical way to lead into the full proof of the sum of an AP. Students will need to know the proof and derivation of the formula for the sum of an arithmetic sequence. • Illustrate how arithmetic sequences are different to geometric sequences, and explain that the common difference (*a*) becomes the common ratio (*r*). Students need to be aware that not all geometric sequences converge. • Cover problems where the *n* in the *n*th term formula (ar^{n-1}) is to be found using logarithms. (Show that it works if we use either base 10 or e.) • Illustrate when to use $\frac{a(1-r^n)}{(1-r)}$ and when to use $\frac{a(r^n-1)}{(r-1)}$ (depending on the value of *r*). • Show that $\frac{a}{(1-r)}$ can be derived if we illustrate on a calculator that r^n tends to zero when $-1 < r < 1$. • A way of illustrating the sum to infinity is to imagine hammering in a nail into a piece of wood, where each strike makes the nail sink in exactly half its remaining distance. There will be a limit to how many times it will need to be hit, as it surely will end up being 'flush' to the surface of the wood and have a distance of zero above the wood. (You can link this to Zeno's paradox.)

Potential Barriers to Access /Misconceptions	**Opportunities for Reasoning/Problem Solving/Proofs**
• When working with formulae for sequences and series, it is important that students state the relevant formula before substituting so that method marks can be awarded even if there is a numerical slip. • Logarithms and exponentials: Compare the geometric sequence $u_n = 2^n$ and the exponential function $f(x) = 2^x$. • Algebra: The binomial theorem starts with $(1-x)^{-1}$, the geometric series starts with $1 + x + x^2 + ..$ and both arrive at the same result. • Using *n* rather than $n - 1$ in the formulae for *n*th term in both arithmetic and geometric sequences. • Mixing up the formulae for *n*th term and the sum of n terms. • Weak algebra. For example, inability to simplify $\frac{32(1.25^n-1)}{1.25-1}$ or to solve $1 - r^3 = 0.488$. • Errors using logarithms in summing geometric series; e.g. in solving $1 - \left(\frac{7}{8}\right)^n > 0.9$	• This topic can be linked to mechanics by investigating, for example, a ball which is dropped from 2 m and bounces to $\frac{3}{4}$ of its height after each bounce. • Challenge students to come up with a rule to determine which series will have a sum to infinity and which won't. • Prove that the infinite arithmetic sequence 3,7,11,15,19,... contains no square numbers. • Prove the formulae for the sum of arithmetic and geometric series **Questions & Prompts** • Explain connections between infinite geometric series and recurring decimals. • Explain connections between the area of a trapezium and summing arithmetic series. • Give me an example of an infinite geometric series with sum 4…and another…and another. • Prove that, for every triangular number T, $8T + 1$ is a square number.

Key Mathematical Vocabulary	Sequence, series, finite, infinite, summation notation, Σ(sigma), periodicity, convergent, divergent, natural numbers, arithmetic series, arithmetic progression (AP), common difference, geometric series, geometric progression (GP), common ratio, *n*th term, sum to *n* terms, sum to infinity (S_∞), limit.

Personal Development	Notes	Resources
Pupils are taught that they must show 'compassion' when working collaboratively as peer support may be required by those potentially making mistakes.		