THE ACADEMY OF ST NICHOLAS	**Y13 Pure**	**CH02** 2.5,2.6,2.7	**Functions & Graphs Transformations & Modelling**	**Lessons 5**

Essential Knowledge Milestones

- understand the effect of simple transformations on the graph of *y* = f(x) including sketching associated graphs and *combinations* of the transformations: *y* = af(x), *y* = f(x) + *a*, *y* = f(x + *a*), *y* = f(*a*x);
- be able to transform graphs to produce other graphs;
- understand the effect of composite transformations on equations of curves and be able to describe them geometrically.
- use functions in modelling, including consideration of limitations and refinements of the models.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: function notation, composites and inverses
- Graphs and transformations and modulus
- Logarithms and exponential - being able to sketch the graphs
- Parametric equations: here functions map a real number into a point in the plane, for example $f(t) = (\cos t, \sin t)$, $0 \leq t < 2\pi$.
- Trigonometry: Restricting the domain of the trig functions for the inverses to exist

Teaching Points

- Students should have some understanding of graph transformations from GCSE (9-1) Mathematics and AS Mathematics – Pure Mathematics, but this will not necessarily include combinations of transformations.
- Students need to be able to sketch the transformations *y* = *a*f(*x*) + *b*, *a*f(*x* + *b*) and f(*ax*) + *b*, but will not be required to sketch f(*ax* + *b*)
- Use graph drawing packages to investigate the properties of familiar functions (such as trigonometric and exponential functions) when you apply the above transformations. Relate the geometry of the transformation to the algebra. For example, f(x) + *a* adds *a* to all the *y*-coordinates, hence the graph moves 'up' by *a* units (translation vector).
- Pose the question, "Does the order in which transformations are applied matter?" Ask students to explore this and present their findings to the class.

The specification gives some possible contexts in which functions can be used to model real-life situations. These are:

- Use of trigonometric functions for modelling tides, hours of sunlight, etc.
- Use of exponential functions for growth and decay
- Use of reciprocal function for inverse proportion (e.g. Pressure and volume)

Potential Barriers to Access/Misconceptions

- Students often score well on questions which involve describing geometrical transformations, but incorrect use of terminology will lose marks. Students must use the correct terms: stretch, scale factor and translation.
- Students also need to be aware that the order of transformations is often important

Opportunities for Reasoning/Problem Solving/Proofs

- Students can explore the difference between transforming x before it goes through the function and transforming it afterwards.
- Prove that the graphs of a function and its inverse are reflections in the line $y = x$
- Prove that if the graphs of $y = f(x)$ and $y = g(x)$ have rotational symmetry about the origin then so has the graph of $y = fg(x)$.

Questions & Prompts

- For any function, $f(x)$, is it ever false that roots of the equation $f(x) = f^{-1}(x)$ lie on the line $y = x$?
- What is the same and what is different about the equations $|ax + b| = x + 3$ and $(ax + b)^2 = (x + 3)^2$?
- Give me three different examples of functions for which $f(-x) = -f(x)$ for all x in the domain. What is the geometrical significance of this property?

Key Mathematical Vocabulary	Function, mapping, domain, range, modulus, transformation, composite, inverse, one to one, many to one, mappings, f(x), fg(x), $f^{-1}x$, reflect, translate, stretch.

Personal Development	**Notes**	**Resources**
Pupils are taught that they must be honest and 'truthful' when feeding back opinions and 'respect' the views of others when discussing the math's techniques used.		