THE ACADEMY OF ST NICHOLAS	Y13 Pure	CH01 2.2,2.3,2.4, 2.5,	Functions & Graphs Composite and Mapping	Lessons 4

Essential Knowledge Milestones

- be able to work out the domain and range of functions;
- know the definition of a one-one and a many-one mappings;
- be able to work out the composition of two functions;
- be able to work out the inverse of a function and sketch its graph;
- understand the condition for an inverse function to exist.

Assumed Prior Knowledge/ Links / Interleaving

- GCSE: function notation, composites and inverses

Potential Barriers to Access /Misconceptions

- Students can often successfully find the range in exam questions, but some give their answer in terms of x rather than f(x).
- When finding inverse functions, students need to remember to swap *x* and *y*. When describing why a function does not have an inverse, students should be advised to answer this question as "because it is not one to one" or "because it is many to one".
- Not restricting the original domain to a one-to-one mapping to establish the inverse function; e.g. in sketching $y = \frac{1}{2}arcsin(x-1)$
- Confusing $sin^{-1}x$ and $(sin\,x)^{-1}$, and more generally $f^{-1}(x)$, $\frac{1}{f(x)}$, $f\ '(x)$

Questions & Prompts

1. Give students two functions to explore.
   For example f(x) = $\frac{1}{x}$ and g(x) = x + 1 or f(x) = |x| and g(x) = x – 2 or f(x) = e^x and g(x) = 2x –1
   Students should explore the following using graphs etc.
   a) compare and contrast the graphs of fg(x) and gf(x)
   b) work out if there are any one-one functions here
   c) find the inverses of any one-one functions (relating the inverses to the originals by sketching)
2. Students should investigate whether the following properties of functions are sometimes true, never true or always true.
   fg(x) = gf(x) g(x) = g^{-1}(x) $(fg)^{-1}$(x) = $g^{-1}f^{-1}$(x) $(fg)^{-1}$(x) = $f^{-1}g^{-1}$(x)

An extension activity could be to find as many functions as possible such that fg(x) = gf(x).

Teaching Points

- The notation f: x ↦… and f(x) will be used as in GCSE (9-1) Mathematics.
- Students will need to understand exactly what functions are and the notation associated with them.
- Domain and range from ℝ (or a subset of ℝ) to ℝ are important terms for students to understand and should be used regularly. Link this to function machines and graphs (where the domain is the set of *x*-values and the range is the set of corresponding *y*-values).
- Students should be aware of one-one and many-one mappings and know that a function cannot be one-many.
- Definitions and examples of odd and even functions will need to be given
- Students need to know how to find the inverse of a function and it is worth stressing the notation here as lots of students still differentiate when they see this in an exam.
- Students should know that if f^{-1} exists, then $ff^{-1}(x) = f^{-1}f(x) = x$. It follows from this that the inverse of a many-one function can only exist if its domain is restricted to make it a one-one function.
- Composite functions are also introduced here and it is worth spending some time going over why the order is very important. Students must know that fg means 'do g first and then f". It may be helpful to use an an additional set of brackets in the notation for composite functions, e.g. f[g(x)].
- Draw lots of examples of the above using graphing packages and relate the mappings to the graphs. Give an example of a quadratic in which the range is determined by the minimum or maximum point.
- Students must also know that the graph of $f^{-1}(x)$ is the image of the graph of *y* = f(x) after reflection in the line *y* = x. You could relate this to the reverse function machine and the algebraic approach for finding an inverse function (when you change the subject of the formula and rewrite it in terms of x as the final step).

Opportunities for Reasoning/Problem Solving/Proofs

- When does the function machine fail to find an inverse?
- Do any functions have a self-inverse?
- Is an inverse function always possible?

Key Mathematical Vocabulary	Function, mapping, domain, range, modulus, transformation, composite, inverse, one to one, many to one, mappings, f(x), fg(x), $f^{-1}x$, reflect, translate, stretch.	
Personal Development	**Notes**	**Resources**
Pupils are taught that they must be honest and 'truthful' when feeding back opinions and 'respect' the views of others when discussing the math's techniques used.		