THE ACADEMY OF ST NICHOLAS	**Y13 Pure**	**CH01** 1.1	**Algebraic Methods:** **Proofs**	**Lessons** **2**

Essential Knowledge Milestones	Teaching Points
• understand that various types of proof can be used to give confirmation that previously learnt formulae are true, and have a sound mathematical basis; • understand that there are different types of proof and disproof (e.g. deduction and contradiction), and know when it is appropriate to use which particular method; • be able to use an appropriate proof within other areas of the specification later in the course. **Assumed Prior Knowledge/ Links / Interleaving** • GCSE: an appreciation of prime factorisation • Problem solving (AS): this introduces other types of proof • Surds and Indices (AS): rational and irrational numbers, manipulating surds • Surds and indices (AS): in this proof unit they now see that operations on surds resulted in a simplest form – surds cannot be expressed as fractions • Algebraic manipulation including completing the square	Explain how verification for a set number of values in *not* a proof of a general result (for all values of n). Show how different methods can be used to prove a statement, including: • Manipulating the LHS of a result and using logical steps (normally algebraic) to make it match the RHS or vice versa (or, sometimes, manipulating both sides to reach the same expression). • Manipulating an expression to show it holds true for all values. For example, an inequality can always be ≥ 0 if we manipulate the LHS to be in the form of [something]^2 since anything squared will always be bigger or equal to zero. This argument can be used on a gradient function to prove a function is increasing. Provide standard examples of proof by contradiction, e.g., $\sqrt{2}$ is irrational: Assuming it can be written as a rational number $\frac{a}{b}$ which has been written in its lowest terms. It follows that $\frac{a^2}{b^2} = 2$ and $a^2 = 2b^2$. Therefore, a^2 is even because it is equal to $2b^2$. It follows that a must be even (as squares of odd integers are never even). Because a is even, there exists an integer k that fulfills: $a = 2k$. Substituting $2k$ for a above gives $2b^2 = (2k)^2 = 4k^2$, so $b^2 = 2k^2$. Because $2k^2 = b^2$, it follows that b^2 is even and b is also even. Hence a and b are both even, which contradicts that $\frac{a}{b}$ is in its simplest form Illustrate proof by exhaustion e.g. Prove that $1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$ for the positive integers from 1 to 5 inclusive. This can be proved if you substitute (exhaust) all the possible values of n from 1 to 5. Note that this type of proof can only be used for proving something for a set of given values. You should also talk about disproof by counter-example. Explain that all we have to do is find *one* example where the statement does not hold and this is enough to show that it is not always true. This method can be used to disprove trigonometric identities as well as statements such as $a > b \Rightarrow a^2 > b^2$: Choose any pair of negative numbers with $a > b$ e.g. $a = -2$ and $b = -3$. Hence $a > b$, but if we square the numbers $a^2 < b^2$ (as 4 < 9) and so this disproves the statement.

Potential Barriers to Access /Misconceptions

- Some students mistakenly think that substituting several values into an expression is sufficient to prove the statement for all values.
- Similarly, for example, referring to a graph to prove that the gradient is always positive rather than completing the square will not gain marks for a proof.
- If a question asks for a particular result to be proved or verified then an appropriate concluding statement is usually required; a ‘tick’ or QED is insufficient.
- Each line of a proof needs to be mathematically correct in order to earn full marks, and the expectations on presentation of argument are higher than at AS.
- Students lose their grip on what they have assumed and what they are trying to prove part way through the proof

Opportunities for Reasoning/Problem Solving/Proofs

- How can we be sure that there is no biggest prime number?
- Explain to me the structure of a proof by contradiction.
- Think of some mathematical truths you already know that can be proved by contradiction

Questions & Prompts

- Prove that there is no smallest positive rational number
- Adapt the proof of the irrationality of $\sqrt{2}$ to prove the irrationality of $\sqrt{5}$ and of $\sqrt[3]{9}$
- Prove that $\sqrt{2} + \sqrt{3}$ is irrational
- Prove that the sum of a rational and an irrational number cannot be rational

Key Mathematical Vocabulary	Proof, verify, deduction, contradict, rational, irrational, square, root, prime, infinity, square number, quadratic, expansion, trigonometry, Pythagoras.

Personal Development	**Notes**	**Resources**
Pupils are taught that they must ‘respect’ each other’s opinions and well-being when working collectively in class. Pupils to learn that mathematicians have ‘ambition’ to push boundaries when aiming to solve new problems.		