| THE ACADEMY OF ST NICHOLAS | Y13 Pure | CH01<br>,1.2,1.3,1.4, 1.5, | Algebraic Methods:<br>Algebraic Fractions, Partial Fractions, Algebraic Division | Lessons<br>4 |
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**Essential Knowledge Milestones**

- be able to add, subtract, multiply and divide algebraic fractions;
- know how to use the factor theorem to shown a linear expression of the form $(a + bx)$ is a factor of a polynomial;
- know how to use the factor theorem for divisors of the form $(a + bx)$;
- be able to simplify algebraic fractions by fully factorising polynomials up to cubic.
- be able to split a proper fraction into partial fractions;
- be able to split an improper fraction into partial fractions, dividing the numerator by the denominator (by polynomial long division or by inspection).

**Assumed Prior Knowledge/ Links / Interleaving**

- Algebraic fractions
- Algebraic division, factor theorem

**Potential Barriers to Access /Misconceptions**

- Students need to practise factorising quadratics as this is often done incorrectly.
- The most common errors include failing to include all necessary brackets, casual miswriting of signs part way through calculations and not dealing correctly with factors. Particular care with signs needs to be taken when a fraction follows a minus sign.
- Some students will set up and solve simultaneous equations rather than using values of x to work out missing constants.
- Ensure students are aware of the most efficient methods for solving different types of problem so they do not waste time in exam situations.

**Teaching Points**

- Revise the basic rules of numerical fractions and start with simplifying some algebraic fractions.
- Exam questions tend to focus on factorising polynomials and then cancelling common factors to simplify algebraic fractions. For example:
  - Simplify $\frac{x^2-5x-6}{x^2-10x+24} \div \frac{x^2-x-2}{x^2-4x}$
- Use function notation when referring to fractions.

  The function f is defined by

  $$f: x \to \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, \ x > \frac{1}{2}$$

  Show that $f(x) = \frac{1}{2x-1}$

Stress the fact that when we break-up a fraction into two or more partial fractions, we use an identity (≡) sign, and not an equal sign, as the expressions are equivalent for all values of x.

The number of partial fractions and the format of the individual terms, is dependent on two factors.

1. The maximum power (or degree) of the polynomials of the numerator and denominator. The degree of the denominator must be *greater* than that of the numerator. If the degree is equal or the degree of the numerator is greater (i.e. the fraction is improper), then algebraic division must be carried out first, and then the partial fractions formed.
2. The type and power of denominator.

   If the denominator is, e.g. $(x + 2)^2$, then we call this a *repeated* factor. In order to cover all possibilities of factors this has to be set up as two partial fractions with denominators $(x + 2)$ and $(x + 2)^2$.

   Show a numerical example with a denominator of 25, and hence the denominators of the partial fractions are 5 and 25.)

Examples of each of the following types need to be covered.

Linear: $\frac{5x-5}{(x+3)(x-2)}$ $\frac{2}{x^2-1}$ $\frac{7x+3}{x(x+1)}$

Repeated: $\frac{4x^2-3x+5}{(x-1)^2(x+2)} \equiv \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x+2)}$

Improper: $\frac{2x^2+5x-6}{(2x-1)(1+x)} \equiv A + \frac{B}{2x-1} + \frac{C}{1+x}$

As students work through examples, encourage them to experiment with the choice of values they substitute. If necessary remind them that x = 0 is an option. Also show that equating coefficients can sometimes be a more efficient alternative, sometimes avoiding the necessity for simultaneous equations.

| Questions & Prompts | Opportunities for Reasoning/Problem Solving/Proofs |
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| • Make up three questions that show you understand different applications of partial fractions. | • showing the reverse process where a simplified rational function is split into two (or more) partial factions.<br>• Are there any values which make the denominators zero? Make links with the graphs of the functions and talk about how these values will correspond to exceptions and special cases in future topics where partial factions need to be found as a simplifying step. |

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| **Key Mathematical Vocabulary** | Polynomial, numerator, denominator, factor, difference of two squares, quadratic, power, index, coefficient, degree, squared, coefficients, improper, identity, algebraic fraction, partial fraction, rational. |

| Personal Development | Notes | Resources |
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| Pupils are taught that they must 'respect' each other's opinions and well-being when working collectively in class. Pupils to learn that mathematicians have 'ambition' to push boundaries when aiming to solve new problems. | | |