THE ACADEMY OF ST NICHOLAS	Y13 Applied	CH08 8.3,8.4,8.5	Mechanics Further Kinematics - Variable Acceleration	Lessons 3

Essential Knowledge Milestones	Teaching Points
• be able to extend techniques for motion in 1 dimension to 2 dimensions by using calculus and vector versions of equations for variable force/acceleration problems; • understand the language and notation of kinematics appropriate to variable motion in 2 dimensions, i.e. knowing the notation $\dot{\boldsymbol{r}}$ and $\ddot{\boldsymbol{r}}$ for variable acceleration in terms of time. **Success Criteria** ❑ You can use calculus with harder functions of time involving variable acceleration Stop. Full stop ❑ You can differentiate and integrate vectors with respect to time. **Assumed Prior Knowledge/ Links / Interleaving** <u>GCSE (9-1) in Mathematics at Higher Tier</u> • Basic trigonometry, Pythagoras and vectors • Find the magnitude and direction of vectors <u>AS Mathematics – Mechanics</u> • Kinematics 1 and equations of motion • Kinematics 2 (variable force) <u>AS Mathematics – Pure</u> • 2D vectors – **i**, **j** system	• This topic links directly to, and is an extension of AS Mathematics – Mechanics content (see SoW Unit 9), which used: ○ $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ and $s = \int v\,dt$, $v = \int a\,dt$ • to model the rates of change for motion of a particle subject to a variable force. • Motions can now be more complicated as the forces in the **i** and **j** directions can differ and be variable (i.e. $\boldsymbol{F} = m\boldsymbol{a}$). Also the notation for 2D motion replaces the displacement, s, with position vector, $\boldsymbol{r}$. Velocity, $\boldsymbol{v}$, can be defined as $\dot{\boldsymbol{r}}$ and the acceleration vector can be called $\ddot{\boldsymbol{r}}$ (rather than $\boldsymbol{a}$). • Introduce this notation to students, explaining how the dot above the $\boldsymbol{r}$ denotes how many times the $\boldsymbol{r}$ has been differentiated with respect to time. Hence $\ddot{\boldsymbol{r}}$ (representing the acceleration) effectively means $\boldsymbol{r}$ differentiated *twice* with respect to time or $\frac{d^2r}{dt^2}$. • The other vital point to stress is when we integrate $\dot{\boldsymbol{r}}$ (or $\boldsymbol{v}$) to obtain the displacement $\boldsymbol{r}$, we have to introduce a vector constant of integration in the form $c\mathbf{i} + k\mathbf{j}$ (rather than just + c). Any conditions provided in the question (e.g. the particle is initially at the point with position vector (3**i** + 2**j**) m) allow us to substitute into the expression for $\boldsymbol{r}$ and calculate the constants. • Ask questions along the lines of: • Consider an aeroplane taking off. Its position is given by $\boldsymbol{r} = (80t\,\mathbf{i} + 0.5t^3\mathbf{j})$ m. What is its velocity and acceleration at time t? Now criticise the model. (Hint: consider motion in the x-direction) • Reverse the process: a particle has acceleration $\boldsymbol{a} = (4t\mathbf{i} + 2\mathbf{j})$ m s^{-2} and is initially at the origin moving with velocity 2**i** m s^{-1}. Find $\dot{\boldsymbol{r}}$ and **r** using integration. (Be careful with the constants of integration!) • Just as in the 1-dimensional case, we do not need to use calculus every time; if the acceleration vector is constant, we can use vector forms of the *suvat* formulae as in Unit 8a. • Questions on this topic often ask about the direction of motion: stress that this is given by the direction of the velocity vector. To find when an object is moving due North, the East component of the velocity vector is zero and the North component positive. • Finally, a question may ask for the force acting on the particle of mass m kg. In this situation students will need to find the acceleration ($\ddot{\boldsymbol{r}}$) at time t and then state the force **F** as $\boldsymbol{F} = m\ddot{\boldsymbol{r}}$ or $\boldsymbol{F} = m\boldsymbol{a}$ (in terms of **i** and **j**).

Potential Barriers to Access /Misconceptions	Opportunities for Reasoning/Problem Solving/Proofs
The following diagram may help students decide whether to differentiate or integrate to solve a problem. '**d**' for the **d**own arrow means '**d**ifferentiate'. Hence, down from $\boldsymbol{r}$ gives $\boldsymbol{v}$ or $\dot{\boldsymbol{r}}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = v$. Integration is the opposite of differentiation so up is integrate. Up from $\boldsymbol{a}(\ddot{\boldsymbol{r}})$ gives $v(\dot{\boldsymbol{r}})$ or integral of $\boldsymbol{a}(\ddot{\boldsymbol{r}})$ with respect to t gives $\boldsymbol{v}(\dot{\boldsymbol{r}})$. ↓ $\boldsymbol{s}$ **d**iff ↓ $\boldsymbol{v}$ ($\dot{\boldsymbol{r}}$) ↑ int $\boldsymbol{a}$ ($\ddot{\boldsymbol{r}}$) ↑	• Some common errors students make include: forgetting the constant of integration; giving the final answer as a vector when the question asked for the speed; and not being careful about changes of direction and so, for example, finding the displacement rather than the distance travelled.

Key Mathematical Vocabulary	Distance, displacement, speed, velocity, constant acceleration, constant force, variable force, variable acceleration, retardation, deceleration, initial (t = 0), stationary (speed = 0), at rest (speed = 0), instantaneously, differentiate, integrate, turning point.

Personal Development	Notes	Resources
Pupils to learn that mathematicians achieve their 'personal best' through working collaboratively with other specialist mathematicians in order to interrelate more than one discipline.		