	Y13 Pure	**CH11** 11.5,		**Integration By Substitution**	**Lessons 3**
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Essential Knowledge Milestones	**Teaching Points**
- be able to integrate expressions using an appropriate substitution; - be able to select the correct substitution and justify their choices.	Most students find integration by substitution challenging and will need to complete lots of different styles of questions. It is a good idea to start with an example which can be performed by inspection as the reverse of differentiation. Students also like to have a step by step process. - Use the given substitution or decide on your own. The substitution is usually the contents of a bracket, square root or the 'nasty' bit! i.e. Let u = … - Differentiate the substitution i.e. $\frac{du}{dx}$ = … - Make dx the subject of the formula - Replace the dx and make the substitution into the integrand - Cancel out any remaining x* - Integrate the resulting (simpler) integral - Substitute back to get the answer in terms of x again *If there are any remaining x, you can re-use the substitution making the x the subject

Assumed Prior Knowledge/ Links / Interleaving	
- AS: Knowledge of e^x and ln x - AS: Laws of logarithms - AS: Trigonometry - AS: Differentiation and integration	

Potential Barriers to Access/Misconceptions	For expressions including trigonometric functions, the identities involving $\sin^2 x$, $\sec^2 x$ are often useful to simplify the integrand.
- Mistakes students make when attempting to integrate by substitution include not changing the dx correctly and simply writing it as du, and failing to substitute back to give an expression in x at the end. - When using integration by substitution, forgetting to change the limits in definite integrals - Inability to select the appropriate method (parts, substitution) for integrating a function - The omission of dx in the student's original integrand and hence the failure to substitute for du.	

Questions & Prompts	**Opportunities for Reasoning/Problem Solving/Proofs**
Show me two ways of finding the indefinite integral $\int \sin x \cos x \, dx$	Try to encourage students to experiment with different substitutions, particularly types involving expressions such as $\sqrt{3x + 4}$. Do we use u^2 = 3x+ 4 or u = 3x+4? The former will require implicit differentiation.

Key Mathematical Vocabulary	Integral, inverse, differential, coefficient, index, power, negative, reciprocal, natural logarithm, ln |x|, coefficient, exponential, identity, sin, cos, tan, sec, cosec, cot, e^x.

Personal Development	**Notes**	**Resources**
Pupils are taught that they must be honest and 'truthful' when feeding back opinions and 'respect' the views of others when discussing the math's techniques used.		