Mathematics	Y11 Higher	Chapter 19 (A7, A12, A13, A14, A15, A21, R7, R10, R11, R13, R14, R15, R16)	The Bigger Picture: Proportion and graphs	Lessons 25

Essential Knowledge	Teaching Points

• Recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$
• State the value of x for which the equation is not defined;
• Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k and integer values of x;
• Use calculators to explore exponential growth and decay;
• Set up, solve and interpret the answers in growth and decay problems;
• Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of f(x) + a, or f(x – a):
  • apply to the graph of y = f(x) the transformations y = –f(x), y = f(–x) for linear, quadratic, cubic functions;
  • apply to the graph of y = f(x) the transformations y = f(x) + a, y = f(x + a) for linear, quadratic, cubic functions;
• Estimate area under a quadratic or other graph by dividing it into trapezia;
• Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
• Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
  • for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  • for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
• Interpret the gradient of a linear or non-linear graph in financial contexts;
• Interpret the area under a linear or non-linear graph in real-life contexts;
• Interpret the rate of change of graphs of containers filling and emptying;
• Interpret the rate of change of unit price in price graphs.
• Recognise and interpret graphs showing direct and inverse proportion;
• Identify direct proportion from a table of values, by comparing ratios of values, for x squared and x cubed relationships;
	• Formal function notation along with inverse and composite functions will have been encountered but are topics that students may need to be reminded about.
	• Financial contexts could include percentage or growth rate.
	• Emphasise when interpreting rates of change with graphs of containers filling and emptying, a steeper gradient means a faster rate of change.
	• When interpreting rates of change of unit price in price graphs, a steeper graph means larger unit price.
	• Translations and reflections of functions are included in this specification, but not rotations or stretches.
	• Students must understand that when two quantities are in direct proportion, the ratio between them remains constant.
	• Students need to know the symbol for 'is proportional to'.
	• Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.

• Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity; • Set up and use equations to solve word and other problems involving direct proportion; • Use $y = kx$ to solve direct proportion problems, including questions where students find k, and then use k to find another value; • Solve problems involving inverse proportion using graphs by plotting and reading values from graphs; • Solve problems involving inverse proportionality; • Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.	
Assumed Prior Knowledge/ Links / Interleaving	
• Students should be able to draw linear and quadratic graphs. • Students should be able to calculate the gradient of a linear function between two points. • Students should recall transformations of trigonometric functions. • Students should have knowledge of writing statements of direct proportion and forming an equation to find values.	
Potential Barriers to Access /Misconceptions	**Opportunities for Reasoning/Problem Solving/Proofs**
• The effects of transforming functions is often confused. • Direct and inverse proportion can get mixed up	• Explain why you cannot find the area under a reciprocal or tan graph. • Interpreting many of these graphs in relation to their specific contexts. • Justify and infer relationships in real-life scenarios to direct and inverse proportion such as ice cream sales and sunshine.
Key Mathematical Vocabulary	Reciprocal, linear, gradient, quadratic, exponential, functions, direct, indirect, proportion, estimate, area, rate of change, distance, time, velocity, transformations, cubic, transformation, constant of proportionality